# Printable Topics in Mathematics

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## Underlying Mathematical Calculations

The key mathematical step in constructing this box is solving the following problem: *Given a spot on the table, where should a bit of plastic be printed on a box face to produce a shadow on the spot?* Formalizing this, suppose the table is the plane $\{(x, y, 0)\}$, the box's face is in the plane $\{(1, y, z)\}$ our light is at $(0, 0, z)$. Then given a spot on the table $(x, y, 0)$, the key observation is that printing plastic anywhere on the line segment between the light and the spot will cast a shadow on the spot. Mathematically, this line segment is given by all the points, for $\lambda \in (0, 1)$,

$$\lambda(x, y, 0) + (1 - \lambda)(0, 0, z)$$

As $\lambda$ approaches zero, we approach the light. As it approaches one, we approach the spot. Picking $\lambda = 1/x$ gives a point on the box's face (checking this is a good exercise!). As a result, we can plug $\lambda$ in to get our answer: To cover $(x, y, 0)$ with a shadow, plastic is needed at

$$\left(1, \frac{y}{x}, \left(1 - \frac{1}{x}\right) z\right) .$$

### *3D Model Viewer and Links*

- `.stl` files and 3D model viewer at `printables.com/model/333924`
- This `.pdf` is available at `ams.jhu.edu/~grimmer/hexBox.pdf`

## Design Process

We arbitrarily chose the height of the box to be the natural number $e$ and the base of our prism was chosen due to corresponding easily to the length properties of a hexagon, $\frac{3}{2}$ and $\sqrt{3}$. After we chose the dimensions of the object, we started determining the coordinates of the shape to project onto the surface via a light. Next, we determined the position of the corners located on the surface of our box by fixing the light source to the point $(0, 0, 0)$ and solving a linear equation, since a location on a line between two points is determined by a simple weightage of the endpoints. Afterward, we utilized the Mathematica software to plot the points and connect their corresponding edges.

# A Freshman Experience Project

# Designing a Box with a Hexagonally Tiling Shadow

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Shadows are a unique way to reduce the dimensionality of an object. The idea of a shadow is projecting a higher dimension surface onto a lower dimension one. For example, the shadow of a cube would result in a 2D polygonal shadow. Likewise, it is possible to think of the shadow of a line as the projection of one vector onto another. In the third dimension, shadows stretch distances or length values but retain properties on angles. And similarly, the shadow of a fourth dimension object should be three-dimensional. Though shadows cannot uniquely define an object, they represent a unique way to think about objects and their properties. Creating shadows for objects in a higher dimension can help us discover unique properties by investigating the properties shadows retain in a lower dimension.

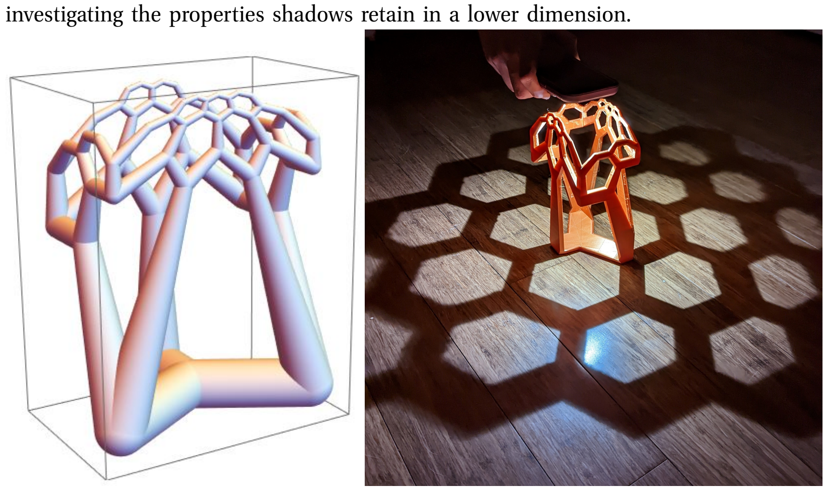

*Figure 1.* Computed model and our final project's lit up shadow.

Our final project combined the different concepts we learned during the semester as it culminating in creating a symmetrical 3D polyhedra that created a hexagonal shadow pattern. We started the semester by viewing the different 3D objects that could be eclipsed by themselves. For example, you can eclipse two cubes of the same size if you rotate a cube 90 degrees counterclockwise and place it directly in front of the other cube, with the first cube completely surrounded by the other. We later focused on the symmetry of different 3D figures and categorized these figures into their degrees, the number of ways the figure could be rotated to produce a symmetrical figure. A bottle has an infinite number of degrees since symmetry exists for it no matter how it is rotated, however, a cube's symmetry is dependent on the angle at which it is looked at. The last major concept we focused on was how the location and angle of a light source impact the shadow that is produced by the object. We learned about parallel, stereographic, and perspective projections and used those ideas in our final project.

A potential alternative to our project could be to keep the same overall structure of the object but to create a closed object rather than one with openings in between each rotation. One way producing a closed object could be beneficial is that the object could have applications such as acting as a cardholder or pencil holder since there would be no gaps in the structure that would allow those objects to fall out. In addition, if the gaps in the structure turned into a rigid surface, we could have the hexagonal pattern that exists on the upper right and left of the object, go through the sides of the object. In this way, we would be able to cast a hexagonal grid from multiple perspectives rather than simply the perspective with light directly above.