Finally, there is a physical problem that is common to many fields, that is very old, and that has not been solved. It is not the problem of finding new fundamental particles, but something left over from a long time ago – over a hundred years. Nobody in physics has really been able to analyze it mathematically satisfactorily in spite of its importance to the sister sciences. It is the analysis of circulating or turbulent fluids.

R. P. Feynman, Vol. I, section 3-7, "How did it get that way?"
The Feynman Lectures on Physics (Addison Wesley, 1964)

Rather than look at the Hamiltonian we shall "wave our hands," use analogies with simpler systems, draw pictures, and make plausible guesses based on physical intuition to obtain a qualitative picture of the solutions (wave functions). This qualitative approach will prove singularly successful.

R. P. Feynman, Chapter 11, "Superfluidity", in Statistical Mechanics,
(Benjamin-Cummings, 1972)

Introduction

This will be a course in turbulence *theory*. The goal is to obtain a precise physical and mathematical understanding of turbulent phenomena (i.e. experiments!) based on the fluid equation of motion. This is *not* a course on modelling of turbulence, which is the focus of many excellent texts, e.g. Pope, Tennekes & Lumley. Many regard modelling as the main goal of the turbulent theory, e.g.

> "The real challenge, it seems to us, is that no adequate model of tubulence exists today." (Tennekes & Lumley, Page 4, Section 1.2)

Some would say that one cannot claim to understand a problem completely if one cannot successfully model it, i.e. compute its behavior quantitatively. This is a “great truth” in the sense of Bohr:

> “A triviality is a statement whose opposite is false. However, a great truth is a statement whose opposite may well be another great truth.” -N. Bohr

Thus, it can also be said that simply modelling a problem—even with some quantitative success—does not imply a correct understanding. A model may give the right answer for the wrong reasons. Also, successful predictions can often be made based on physical and mathematical understanding, without the use of a “model”. In our opinion, the only truly successful and predictive models of turbulence will require a clear understanding of the physics. Although we shall not pursue modelling in this course, our discussion shall be closely connected with various modelling approaches (particularly LES or Large-Eddy Simulation).

Some other distinctive features of this course:

(1) We shall avoid Fourier space as much as possible! Although spectral methods were useful in the early days of the subject, my own experience is that they are rarely necessary and usually obscure the physics of turbulence. A notable exception is wave dynamics, where some version of Fourier methods is important to proper understanding.

(2) We also avoid probability methods, as far as possible. This is not because probability is unimportant in turbulence—we believe it plays a very fundamental role. However, any purely probabilistic approach to turbulence is inadequate: when you stir cream into your morning coffee, there is only one flow realization and there are no “ensembles”, but there is nevertheless turbulence! Note that the LES method of turbulence modeling deals also with individual realizations of turbulent flows and is not based on statistical averaging. Furthermore, many “applications” of statistical methods to turbulence are completely gratuitous. Indeed, a very great many facts of turbulence can be understood without any probability or statistical averaging. They are purely dynamical results that follow directly from the Navier-Stokes equation—i.e. they hold for every realization with probability one! It is only for certain precise problems, e.g.

predicting the future behavior of a turbulent flow, where statistical methods are essential. We avoid the use of probability wherever possible, so that it is clear where it is really needed.

NOTES & REFERENCES

Tennekes, H. and Lumley, J. H., A first course in turbulence, *The MIT Press*, 1972.

Pope, S. B., Turbulent flows, *Cambridge University Press*, 2000.

Frisch, U., Turbulence, *Cambridge University Press*, 1995.