Homework No.1, 553.793, Due January 31, 2025.

1. Suppose that an automobile with drag coefficient $C_D = 0.4$ wastes 30% of the energy provided by gasoline combustion in turbulent dissipation by air resistance. If C_D is lowered to 0.3 by aerodynamic redesign of the body and if other energy consumption is the same, show that the amount of fuel burned drops by 7.5%. In the United State alone, this would save about 28 million gallons of gasoline per day!

`https://www.eia.gov/tools/faqs/faq.cfm?id=23&t=10`

2. (Tennekes & Lumley, Problem 1.2). A cubical box of volume L^3 is filled with fluid in turbulent motion. No source of energy is present, so that the turbulence decays. Because the turbulence is confined to the box, its length scale may be assumed to be equal to L at all times. Obtain an expression for the decay of the kinetic energy $\frac{3}{2}u^2$ as a function of time. As the turbulence decays, its Reynolds number decreases. If the Reynolds number uL/ν becomes smaller than 10, say, the inviscid estimate $\varepsilon = u^3/L$ should be replaced by an estimate of the type $\varepsilon = c\nu u^2/L^2$, because the weak eddies remaining at low Reynolds numbers lose their energy directly to viscous dissipation. Compute the constant prefactor c by requiring that the dissipation rate is continuous at $uL/\nu = 10$. Derive an expression for the decay of the kinetic energy when $uL/\nu < 10$ (this is called the "final period of decay"). If $L = 1$ m, $\nu = 15 \times 10^{-6}$ m^2/sec and $u = 1$ m/sec at time $t = 0$, how long does it take before the turbulence enters the final period of decay? Assume that the effects of the walls of the box on the decay of the turbulence may be ignored. Can you support this assumption in any way?

Remark: This problem from the book of Tennekes & Lumley assumes that $\varepsilon = Cu^3/L$ with a constant $C = 1$. The experimental data we reviewed showed that C is a constant of order unity, but differing from flow to flow. Results not much different are obtained if one takes another constant $C \neq 1$, but, crucially, independent of dimensonless time $\tau = Ut/L$. We shall return to the assumption of time-independence of C or other "universality assumptions" after we develop some more theory.

3. In turbulent pipe flow, the turbulent velocity scale is set by the *friction velocity* u_* which is defined by the relation $\tau_* = u_*^2$, where τ_* is the viscous shear stress at the pipe wall. In the classical theory of the logarithmic "law of the wall", this is related to the velocity U_0 at the centerline of the pipe by

$$\frac{U_0}{u_*} = 2.5 \ln(Re_*) + 6,$$

where $Re_* = u_* r/\nu$ is the Reynolds number formed by u_* and the radius r of the pipe. The energy dissipation in the central "core" region of the pipe is of the order of $\varepsilon \sim u_*^3/r$. Suppose that the Reynolds number $Re = U_0 r/\nu$ is increased with fixed U_0 and r by decreasing the kinematic viscosity $\nu = \eta/\rho$ of the working fluid (e.g. by increasing the density) from 10^4 to $Re = 10^{10}$. By what factor does the bulk dissipation decrease? You may calculate u_*/U numerically as a function of Re from the above relation by a fixed point method or by using the Lambert W-function

`https://en.wikipedia.org/wiki/Lambert_W_function`

The asymptotic approximation $u_* \approx 0.4 U_0/\ln(Re)$ for $\ln(Re) \gg 1$ gives a reasonable estimate only at extremely high Reynolds numbers.

Note, by the way, that with rough walls of mean roughness height h the empirical relation between u_* and U_0 becomes Reynolds-number independent:

$$\frac{U_0}{u_*} = 2.5 \ln(\frac{r}{h}) + (\text{const.}).$$

Compare this situation with that in the experiment of Cadot et al. (1997).

4. In perfectly still air, aerosols are transported only by molecular diffusion, where their concentration c obeys $\partial_t c = D \triangle c$, with D the *molecular diffusivity.* A realistic value* for a relatively heavy molecule such as a perfume in air at room temperature is $D = 5\,\text{mm}^2/\text{sec}$. If the length of a room is $L = 5\text{m}$, calculate the characteristic time $T = L^2/D$ for the perfume to diffuse across the room. Convert the time in seconds to hours and days.

*For example, see M. Teixeira et al., "The diffusion of perfume mixtures and the odor performance," Chemical Engineering Science 64: 2570–2589 (2009), Table 1.

5. In a cumulus cloud a typical turret (arguably the length scale of the largest eddies) is about 250 m. The updraft velocity is of order 1 m/s and the density $\rho \doteq 1.25\,\text{kg/m}^3$. Estimate the energy dissipation rate per unit mass and the total dissipation rate in kilowatts for one "eddy" of the cloud (assumed to be a sphere). Compare with the power received at the surface of the Earth from the sun, which is about 1 kilowatt per m^2 (on a clear day) and with the power consumption of a 300 horsepower automobile.