Problem 1. (a) Strikwerda, Exercise 4.1.1.
(b) Strikwerda, Exercise 4.1.4.

Problem 2. (a) Write a code to implement the higher-order (2, 4) leapfrog scheme
\[
\frac{v_m^{n+1} - v_m^n}{2k} + a \left( 1 - \frac{h^2 \delta^2}{6} \right) \delta_0 v_m^n = f_m^n.
\]
Perform the first time step with the corresponding improvement of the forward-central scheme:
\[
\frac{v_m^{n+1} - v_m^n}{k} + a \left( 1 - \frac{h^2 \delta^2}{6} \right) \delta_0 v_m^n = f_m^n.
\]
Solve for interior points \( m = 2, \ldots, M - 2 \) using the numerical scheme and, for \( a > 0 \), use the physical boundary conditions at the left boundary points \( m = 0, 1 \) and the quasi-characteristic extrapolations
\[
v_{M-1}^{n+1} = 2v_{M-2}^n - v_{M-3}^n, \quad v_M^{n+1} = 2v_{M-1}^n - v_{M-2}^n,
\]
at the right boundary points \( m = M - 1, M \).
(b) Compare your code in part (a) with standard leapfrog method for the one-way wave equation \( u_t + u_x = 0 \) on the interval \( x \in [-2, 3] \) and for time range \( t \in [0, 1.6] \), with initial condition \( u_0(x) = \exp(-16x^2) \). For both methods, take \( h = 1/20, \lambda = 0.4 \), so that the computational effort is approximately the same. Plot the numerical solutions of both methods together with the exact solution at times \( t = 0.4, 0.8, 1.6 \).
At the final time \( t = 1.6 \) calculate the relative error for both methods in the discrete \( L^2 \)-norm over the entire space interval and also over just the space interval \( [1, 3] \). Which method is more accurate and where?


Problem 4. Strikwerda, Exercise 5.1.2,

Problem 5. (a) Write a code to implement the dissipative Crank-Nicolson scheme
\[
\frac{v_m^{n+1} - v_m^n}{k} + \frac{a}{2} \delta_0 (v_m^{n+1} + v_m^n) + \frac{\varepsilon}{k} \left( \frac{1}{2} h \delta \right)^4 v_m^n = \frac{1}{2} (f_m^{n+1} + f_m^n).
\]
For \( a > 0 \), use the physical boundary conditions at the right end-point \( m = 0 \) and the implicit boundary conditions \( u_M^{n+1} = 2u_{M-1}^{n+1} - u_{M-2}^{n+1} \) at the right end-point \( m = M \). To evaluate the dissipation correction term proportional to \( \varepsilon \) at the interior point \( m = 1 \) use the second-order extrapolation \( u_0^n = 2u_0^n - u_1^n \), and at the interior point \( m = M - 1 \) similarly use \( u_M^n = 2u_M^n - u_{M-1}^n \).
(b) Compare your code in part (a) with standard Crank-Nicolson method for one-way wave equation $u_t + u_x = 0$ on the interval $x \in [-2, 3]$ and for time range $t \in [0, 1.6]$, with initial condition $u_0(x) = 1 - |x|$ for $|x| < 1$ and $= 0$ elsewhere. For both methods, take $h = 1/20, \lambda = 0.8$, so that the computational effort is approximately the same. Take parameter $\varepsilon = 4(a\lambda)^2(1 - (a\lambda)^2)$ in the dissipative Crank-Nicolson scheme. Plot the numerical solutions of both methods together with the exact solution at times $t = 0.4, 0.8, 1.6$. Where do you observe an improvement in the solution?