Homework No.10, 553.391, Due December 6, 2021.

1. Strogatz, Problem 8.7.2.

2. Strogatz, Problem 8.7.3. Hint: It may be helpful to use the well-known formula
   \[ x_p(t) = \int_0^t e^{s-t} F(s) \, ds \]
   for a particular solution of the inhomogeneous linear ODE
   \[ \dot{x} + x = F(t). \]


4. Strogatz, Problem 9.2.2. Hint: If \( V = rx^2 + \sigma y^2 + \sigma (z-2r)^2 \), show that \( dV/dt < 0 \)
   except inside an ellipsoidal region \( E_1 \). Then show that a constant \( C \) exists so large
   that \( V(x,y,z) \leq C \) defines an ellipsoidal region \( E_2 \) strictly containing \( E_1 \).

5. Strogatz, Problem 9.3.1.

6. Strogatz, Problem 9.3.3. Do not make a plot of \( x \) vs. \( z \), as Strogatz requests, but
   instead make 3D plots of \((x, y, z)\). You may use the MATLAB plotting option \texttt{odephas3}
   with integrators like \texttt{ode45} or else use \texttt{plot3} to plot output data arrays. Also, plot
   the time series of all three variables \( x(t), y(t) \) and \( z(t) \).

7. Strogatz, Problem 9.3.4. Same instructions as Problem 4 above.

8. Strogatz, Problem 9.3.9. You can use one of the standard integrators in MATLAB
   such as \texttt{ode45} to obtain a solution matrix \([t_1,y_1]\) for the Lorenz model. Make sure
   that your initial vector is on the attractor! Repeat with another nearby initial vector
   to create a second solution matrix \([t_2,y_2]\). Since the MATLAB integrators like \texttt{ode45}
   do not use a uniform time-step, it requires a little bit of extra work to compare two
   nearby trajectories. Even if one integrates over the same interval of time \([t_0 \, t_f]\),
   the solution output vectors for two nearby initial conditions need not have the same
   length! To make the comparison, it is necessary to interpolate the outputs on a
   uniform grid of times \( tt = t_0 : dt : t_f \), for example, by using spline interpolation in
   MATLAB with the command
   \[
   yy1 = \text{interp1}(t1,y1,tt,'spline'); \quad yy2 = \text{interp1}(t2,y2,tt,'spline');
   \]
   Then type
   \[
   dyy = yy2 - yy1;
   \]
   to create the difference. The command
   \[
   nn = \text{sum(abs(dyy))};
   \]
will create a row vector of norms of the differences for all times. Then

\[
\text{plot}(tt, \log(nn));
\]

will plot the logarithm of the norm versus time, and

\[
\text{polyfit}(tt, \log(nn), 1);
\]

will give the slope and intercept of the best linear fit.

9. Strogatz, Problem 9.4.2.(a),(b),(c). Do not do Strogatz’ part (d). Instead, calculate analytically the exact value of the Lyapunov exponent $\lambda$ of the tent map for each point in the interval $[0, 1]$.

10. Strogatz, Problem 9.5.2. Same instructions as Problem 4 above.