From: Lawrence & Pasternack Applied Management Science 2nd ed

5.1 Introduction to Project Scheduling

We can think of a *project* as a collection of tasks a person or firm desires to complete in minimum time or at minimal cost. For example, in painting the outside of a house a contractor must: (1) select color(s); (2) purchase paint; (3) clean existing siding; (4) mask windows; (5) spray paint large areas; (6) hand paint trim; and (7) clean up. Factors affecting the completion time might include the number of painters the contractor employs, the availability of the individual paint colors, and the square footage and special details of the house. The painting contractor's primary objective might be to finish painting the outside of the house in minimal time so that he can move on to another project, or at minimal total cost so that he will earn the maximum profit from his work.

The tasks of a project are called *activities*. Estimated completion times (and sometimes costs) are associated with each activity of a project. Activities can be defined broadly or narrowly, depending on the situation. For example, an activity involved in bringing a new play to Broadway might be "Hire Cast." Although this description is appropriate for some models, in other cases it might be beneficial to subdivide this part of the project into much narrower sets of activities, such as:

"Hold Auditions for Principal Characters"
"Arrange for Call-Back Auditions of Principal Characters"
"Cast Principal Characters"
"Hold Auditions for Extras"
"Cast Extras"
"Hold Preliminary Run-Through"
"Make Final Cast Selections"

The degree of detail depends on both the application and the level of specificity in the available time and cost data.

In any project, certain activities must be completed before others are started, whereas others may be completed simultaneously. For a Broadway play, "Hire Cast" certainly must precede "Dress Rehearsal," but "Dress Rehearsal" need not precede "Advance Ticket Sales," or vice versa. Determining an accurate set of *precedence relations* among the activities—that is, detailing which activities must precede others—is crucial to developing an optimal schedule for the individual activities.

The completion time for each activity, and, thus, the overall project completion time, is generally related to the amount of resources committed to it. In the opening vignette, the completion time of the Taco Bell restaurant could not have been reduced from its *normal completion time* of almost two months to its *crash completion time* of two days without spending extra money (on overtime wages and talented crews willing to work at night). And even though Taco Bell was committed to meeting a prepublicized deadline of 48 hours, it wished to do so at minimum total cost. Similarly, in the freeway repair problem, since contractors were offered a bonus of $15 million for completing the repairs before a specified date, additional resources were committed to critical activities in the successful effort to meet this deadline. In both cases, the positive results did not just happen. Rather, each success was the result of careful and comprehensive project planning, scheduling, and monitoring.

OBJECTIVES OF PROJECT SCHEDULING

Project scheduling is used to plan and control a project efficiently. Some of the objectives of project scheduling include:

- Determining a schedule of earliest and latest start and finish times for each activity that leads to the earliest completion time for the entire project

- Calculating the likelihood that a project will be completed within a certain time period
- Finding the minimum cost schedule that will complete a project by a certain date
- Investigating how delays to certain activities affect the overall completion time of a project
- Monitoring a project to determine whether it is proceeding on time and within budget
- Finding a schedule of activities that will smooth out the allocation of resources over the duration of the project

The above objectives can be accomplished using project scheduling proaches, such as PERT (Program Evaluation and Review Technique) and (Critical Path Method). These methods, which were both developed in the 1950s, use project networks to help schedule a project's activities. Althoug distinction between the two techniques has blurred in recent years, PERT is erally regarded as a method that treats the completion time of the activities a: dom variables with specific probability distributions, whereas CPM assume the completion time of an activity is solely dependent on the amount of n spent to complete the activity.

Both PERT and CPM require the modeler to identify the activities project and the *precedence relations* between them. This involves determining of *immediate predecessors* for each activity. An activity's immediate predecessc those jobs that must be completed just prior to the activity's commencem **precedence relations chart** identifies the separate activities of the proje their precedence relations. From this chart a PERT/CPM network represen of the project can then be constructed.

5.2 Identifying the Activities of a Project

To illustrate the concepts of project scheduling, consider the situation fa Klone Computer, Inc.

KLONE COMPUTERS, INC.

Klonel.cpm

Klone Computers is a small manufacturer of personal computers which is a design, manufacture, and market the Klonepalm 2000 palmbook comput company faces three major tasks in introducing a new computer: (1) manu ing the new computer; (2) training staff and vendor representatives to ope new computer; and (3) advertising the new computer.

When the proposed specifications for the new computer have been re the manufacturing phase begins with the design of a prototype compute the design is determined, the required materials are purchased and pr manufactured. Prototype models are then tested and analyzed by staff pe who have completed a staff training course. Based on their input, refinem made to the prototype and an initial production run of computers is sched

Staff training of company personnel begins once the computer is desig lowing the staff to test the prototypes once they have been manufacture the computer design has been revised based on staff input, the sales forc goes full-scale training.

Advertising is a two-phase procedure. First, a small group works closely with the design team so that once a product design has been chosen, the marketing team can begin an initial preproduction advertising campaign. Following this initial campaign and completion of the final design revisions, a larger advertising team is introduced to the special features of the computer, and a full-scale advertising program is launched.

The entire project is concluded when the initial production run is completed, the salespersons are trained, and the advertising campaign is underway. As a first step in generating a project schedule, Klone needs to develop a precedence relations chart that gives a concise set of individual tasks for the project and shows which other tasks must be completed prior to the commencement of each task.

SOLUTION

The entire project can be represented by the ten activities—five manufacturing, three training, and two advertising—given in Table 5.1. For easy reference, each activity is designated by a letter symbol.[1] After identifying the activities, we determine the immediate predecessors for each one using the reasoning in Table 5.2.

TABLE 5.1 Klonepalm 2000 Activity Description

	Activity	Description
Manufacturing activities	A	Prototype model design
	B	Purchase of materials
	C	Manufacture of prototype models
	D	Revision of design
	E	Initial production run
Training activities	F	Staff training
	G	Staff input on prototype models
	H	Sales training
Advertising activities	I	Preproduction advertising campaign
	J	Post-redesign advertising campaign

These relations, along with Klone's estimates of the expected completion time for each activity (based on its past experiences manufacturing similar products), are summarized in the *precedence relations chart* shown in Table 5.3. We will use this chart to construct graphical representations of the project.

If each activity of the Klonepalm 2000 project were performed sequentially, ignoring the possibility that some of the activities of the project could be completed simultaneously, the estimated completion time of the project would be $90 + 15 + 5 + 20 + 21 + 25 + 14 + 28 + 30 + 45 = 293$ days. Since work on several of the activities *can* be underway at the same time, however, the time required to complete the project will be less than 293 days. The goal of the management science team is to schedule activities so that the entire project is completed in the minimal number of days.

[1] Instead of letters, we could have used abbreviations or some other identifier. Which letter will designate which activity is a purely arbitrary decision and has no implications for which activity must be completed first.

TABLE 5.2 Immediate Predecessors for Klonepalm 2000 Activities

Activity	Requirements	Immediat Predecesso
A Prototype design	No requirements	—
B Purchase of materials	Materials can be purchased only after the prototypes have been designed (A).	A
C Manufacture of prototypes	Materials must be purchased (B) before the prototypes can be manufactured.	B
D Revision of design	Both prototype manufacturing (C) and staff input (G) must precede the design revision. However, since prototype manufacturing precedes staff input, it is not an *immediate* predecessor.	G
E Initial production run	The production run can begin after the design has been revised (D).	D
F Staff training	Staff training begins after the prototype is designed (A).	A
G Staff input on prototype	For staff input on prototypes, the prototype must be built (C) and the staff trained (F).	C,F
H Sales training	Salespersons can be trained immediately after the design revision (D).	D
I Preproduction advertising campaign	The initial preproduction advertising campaign can begin as soon as the prototypes have been designed (A).	A
J Post-redesign advertising campaign	The large-scale postproduction advertising campaign begins when the initial ad campaign has been completed (I) and the design has been revised (D).	D,I

TABLE 5.3 Klonepalm 2000 Precedence Relations Chart

Activity	Immediate Predecessors	Estimated Completion Time (day
A	—	90
B	A	15
C	B	5
D	G	20
E	D	21
F	A	25
G	C,F	14
H	D	28
I	A	30
J	D,I	45

THE PERT/CPM NETWORK

PERT/CPM is one popular approach used in project scheduling PERT/CPM approach is based on a network representation that reflects precedence relations. As shown in the network in Figure 5.1, the nodes de activities and their time duration, and the arcs define the precedence relati tween the activities. When an activity immediately precedes one or more o tivities, a directed arc is drawn from its node to each node representing on subsequent activities. Note that there is one arc for each distinct entry in mediate predecessor column in Table 5.2.

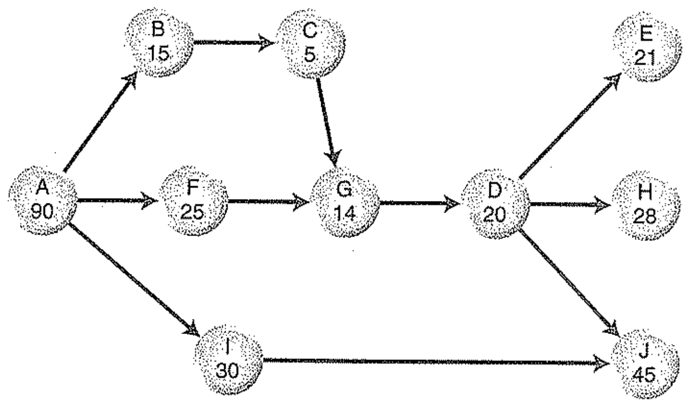

FIGURE 5.1
PERT/CPM Network for the Klonepalm 2000 Project

5.3 The PERT/CPM Approach for Project Scheduling

Two primary objectives of PERT/CPM analyses are (1) to determine the minimal possible completion time for the project; and (2) to determine a range of start and finish times for each activity so that the project can be completed in minimal time. To illustrate how the above network can be used to achieve these objectives, let us return to the planning process faced by Klone Computers.

KLONE COMPUTERS, INC. (CONTINUED)

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Management at Klone Computers would like to schedule the activities of the Klonepalm 2000 project so that it is completed in minimal time. In particular, management wishes to know:

1. The earliest completion date for the project
2. The earliest and latest start times for each activity which will not alter this date
3. The earliest and latest finish times for each activity which will not alter this date
4. The activities that must adhere to a rigid fixed schedule and the activities that can possibly be delayed without affecting the project completion time.

SOLUTION

The PERT/CPM approach for the Klonepalm 2000 computer project is illustrated by referring to the network developed for this problem in Figure 5.1. This is dynamically displayed on the PowerPoint slides on the accompanying CD-ROM.

EARLIEST START/FINISH TIMES—EARLIEST COMPLETION DATE

To determine the **earliest start time (ES)** and the **earliest finish time (EF)** for the activities, a *forward pass* is made through the network. We begin by evaluating all activities that have no immediate predecessors—in this case, only activity A. The ES for an activity with no predecessors is 0; its EF is simply the activity's completion time. Thus, for activity A, ES(A) = 0, EF(A) = 90.

We then proceed by selecting any node for which the EF of all its immediate predecessors has been determined—in this case B, F, and I. Since *all* of an activity's immediate predecessors must be completed before the activity can begin, the

ES for this activity is the *maximum* of the EFs of its immediate predecessors. EF then equals its ES plus the time to complete the activity. Since activities B, and I require only the completion of activity A, and since the activity completi times for activities B, F, and I are 15, 25, and 30, respectively, we conclude:

$$
\begin{array}{ll}
ES(B) = 90 & EF(B) = 90 + 15 = 105 \\
ES(F) = 90 & EF(F) = 90 + 25 = 115 \\
ES(I) = 90 & EF(I) = 90 + 30 = 120
\end{array}
$$

Since activity B is the immediate predecessor for activity C and EF(B) = 1 we now can conclude:

$$ES(C) = 105 \qquad EF(C) = 105 + 5 = 110$$

Now consider activity G. Both activity C (with earliest finish time of 110) activity F (with earliest finish time of 115) are immediate predecessors of activity Since both must be completed before activity G can commence, the earliest s time for activity G is the maximum of the earliest finish times for activities C an

$$ES(G) = MAX(EF(C), EF(F)) = MAX(110, 115) = 115$$

and,

$$EF(G) = 115 + 14 = 129$$

Hence we see that the following relationships exist when calculating ES and EF ti

Earliest Start/Finish Time for an Activity

ES = MAXIMUM EF of all its immediate predecessors

EF = ES + (Activity Completion Time)

We now repeat this process until all nodes have been evaluated. This giv a schedule of earliest start and finish times for each activity. The maximum c EF times of all nodes is the earliest completion time for the project. The seq of calculations that determine these times is given in Table 5.4. **The maximu EF times, 194, is the estimated completion time of the entire project.**

TABLE 5.4 Sequence of ES and EF Calculations

Activity	Immediate Predecessors (EF)	ES	EF
A	—	0	0 + 90 =
B	A (90)	90	90 + 15 =
F	A (90)	90	90 + 25 =
I	A (90)	90	90 + 30 =
C	B (105)	105	105 + 5 =
G	C (110), F (115)	115	115 + 14 =
D	G (129)	129	129 + 20 =
E	D (149)	149	149 + 21 =
H	D (149)	149	149 + 28
J	D (149), I (120)	149	149 + 45

The ES and EF for each activity are represented on the PERT/CPM network by a pair of numbers above the node representing the activity, as shown in Figure 5.2.

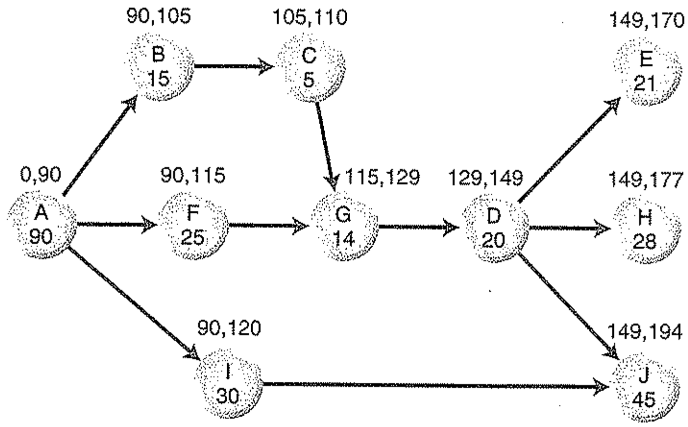


FIGURE 5.2
Earliest Start and Finish Times for the Klonepalm 2000 Project

LATEST START/FINISH TIMES

To determine the **latest start time (LS)** and **latest finish time (LF)** for each activity which allows the project to be completed by its minimal completion date of 194 days, a *backwards pass* is made through the network. We begin by evaluating all activities that have no successor activities. These are activities E, H, and J, which have completion times of 21, 28, and 45, respectively. The LF for each of these activities is the minimal project completion time (194 days). The LS for each of these activities is determined by subtracting the corresponding activity's duration from its LF value. Thus,

$$\begin{aligned} LF(E) &= 194 \qquad & LS(E) &= 194 - 21 = 173 \\ LF(H) &= 194 \qquad & LS(H) &= 194 - 28 = 166 \\ LF(J) &= 194 \qquad & LS(J) &= 194 - 45 = 149 \end{aligned}$$

Continuing the backwards pass through the network, we see that activity J is the only successor activity to activity I. Activity I must therefore be finished in time to start activity by activity J's latest start time:

$$LF(I) = LS(J) = 149$$

Thus,

$$LS(I) = 149 - 30 = 119$$

Activity D, however, is the predecessor to three activities (E, H, and J). Hence it must be finished in time to start each of these activities by their LS times. That is, activity D must be finished in time to start activity E by day 173, in time to start activity H by day 166, and in time to start activity J by day 149. For all three of these conditions to be met, activity D must be finished by day 149, that is,

$$LF(D) = MIN(LS(E), LS(H), LS(J)) = MIN(173, 166, 149) = 149$$

Thus,

$$LS(D) = 149 - 20 = 129$$

From this discussion we see that the following relationships exist for calculat
LF and LS times for any activity.

Latest Start/Finish Times for an Activity

LF = MINIMUM LS of all immediate successor activities

LS = LF − (Activity Completion Time)

We repeat this process until all nodes have been evaluated, as shown in Table !

TABLE 5.5 Sequence of LF and LS Calculations

Activity	Immediate Successors (LS)	LF	LS
J	—	194	194 − 45 =
H	—	194	194 − 28 =
E	—	194	194 − 21 =
I	J (149)	149	149 − 30 =
D	E (173), H (166), J (149)	149	149 − 20 =
G	D (129)	129	129 − 14 =
C	G (115)	115	115 − 5 =
F	G (115)	115	115 − 25 =
B	C (110)	110	110 − 15 =
A	B (95), F (90), I (119)	90	90 − 90 =

We denote the LS and LF for each activity on the PERT/CPM netwc
placing these numbers below the corresponding node representing it. Figure
a complete network representation showing both the earliest and latest sta
finish times for the Klonepalm 2000 project.

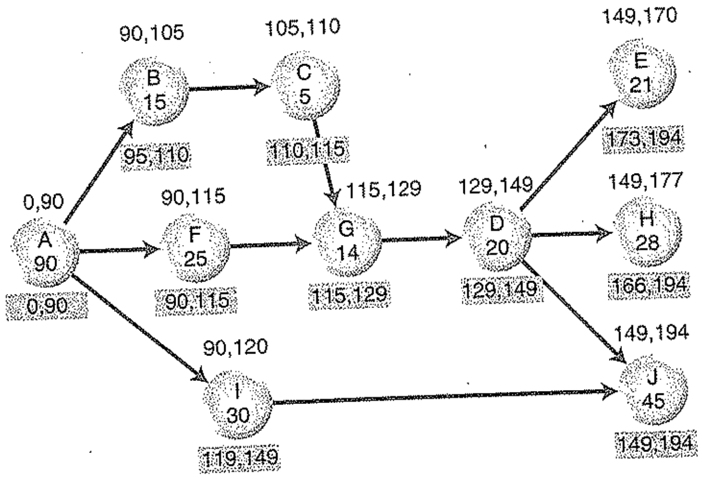


FIGURE 5.3
Earliest/Latest Start and Finish Times for the Klonepalm 2000 Project

THE CRITICAL PATH AND SLACK TIMES

In the course of completing a project, both planned and unforeseen delays
fect activity start or completion times. For example, revising the desig
computer (activity D), which is scheduled to take 20 days, may actually re

days. Or management may have to delay the start of sales training (activity H) by five days because the firm's training classroom might have been previously booked for another function. Some of these delays affect the overall completion date of the project; others may not.

To analyze the impact of such delays on the project, we determine the **slack time** for each activity. Slack time is the amount of time an activity can be delayed from its ES without delaying the project's estimated completion time. It is calculated by subtracting an activity's ES from its LS (or its EF from its LF). This value for an activity's slack time assumes that *only the completion time of this single activity has been changed* and that there are *no other delays to activities in the project*. Table 5.6 details the slack time calculations for each activity in the Klonepalm 2000 project.

TABLE 5.6 Slack Times

Activity A:	Slack (A)	= LS(A)	− ES(A)	=	0	−	0 =	0
Activity B:	Slack (B)	= LS(B)	− ES(B)	=	95	−	90 =	5
Activity C:	Slack (C)	= LS(C)	− ES(C)	=	110	−	105 =	5
Activity D:	Slack (D)	= LS(D)	− ES(D)	=	119	−	119 =	0
Activity E:	Slack (E)	= LS(E)	− ES(E)	=	173	−	149 =	24
Activity F:	Slack (F)	= LS(F)	− ES(F)	=	90	−	90 =	0
Activity G:	Slack (G)	= LS(G)	− ES(G)	=	115	−	115 =	0
Activity H:	Slack (H)	= LS(H)	− ES(H)	=	166	−	149 =	17
Activity I:	Slack (I)	= LS(I)	− ES(I)	=	119	−	90 =	29
Activity J:	Slack (J)	= LS(J)	− ES(J)	=	149	−	149 =	0

When an activity has slack time, the manager has some flexibility in scheduling and may be able to distribute the workload more evenly throughout the project's duration without affecting its overall completion date. This is especially important in projects with limited staff or resources. The concept of **resource leveling** is discussed in more detail in Section 5.7.

Activities that have *no* slack time (activities A, D, F, G, and J) are called **critical activities**. These activities must be rigidly scheduled to start and finish at their specific ES and EF times, respectively. Any delay in completing a critical activity will delay completion time of the entire project beyond 194 days by the corresponding amount.

Slack Time/Critical Activities

Slack time for an activity = LS − ES or LF − EF.

Critical activities are those with slack time = 0.

As Figure 5.4 illustrates, these critical activities form a path, called a **critical path**, through the network. The sum of the completion times of the activities on the critical path is the minimal completion time for the project (90 + 25 + 14 + 20 + 45 = 194). Because it consists of the sequence of activities that cannot be delayed without affecting the earliest project completion date, the critical path is actually the *longest path in the directed network*. Summarizing, for the Klonepalm 2000 project, we have:

EXPECTED PROJECT COMPLETION TIME: 194 days
CRITICAL PATH: A–F–G–D–J

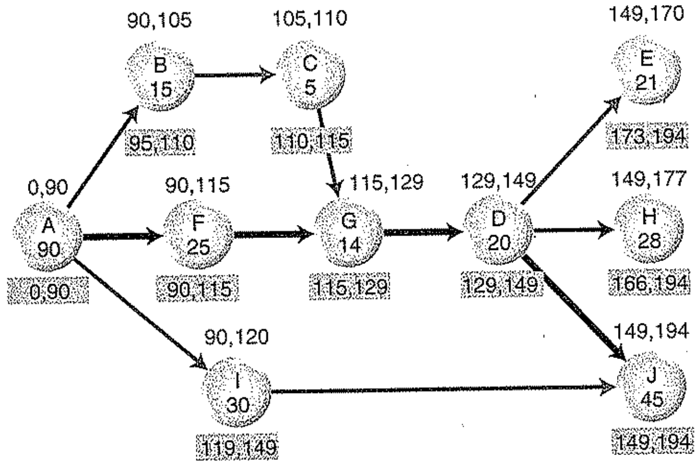


FIGURE 5.4
Critical Path for the Klonepalm 2000 Project: A–F–G–D–J

It is possible to have more than one critical path in a PERT/CPM network. For example, if the completion time of activity I had been 59 days rather than 30 days, both its earliest and latest start times would be 149. Thus, a second critical path giving a total completion time of 194 days would have consisted of activities A, I, and J.

Critical Path

1. The critical activities (activities with 0 slack) form *at least one* critical path in the network.
2. A critical path is the longest path in the network.
3. The sum of the completion times for the activities on the critical path gives the minimal completion time of the project.

ANALYSIS OF POSSIBLE DELAYS

The ES, EF, LS, LF, and slack for each activity are frequently condensed into a single chart, known as an *activity schedule chart* (see Table 5.7).

TABLE 5.7 Activity Schedule Chart: Klonepalm 2000 Project

Activity	ES	EF	LS	LF	Slack
A	0	90	0	90	0
B	90	105	95	110	5
C	105	110	110	115	5
D	129	149	129	149	0
E	149	170	173	194	24
F	90	115	90	115	0
G	115	129	115	129	0
H	149	177	166	194	17
I	90	120	119	149	29
J	149	194	149	194	0

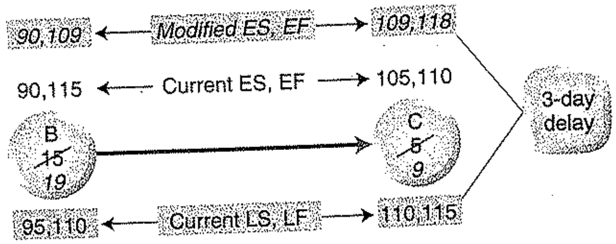


FIGURE 5.5
Case 3: Multiple Delays along the Critical Path

by a critical activity. In both cases, the overall project completion date does 1 change.

In Case 3, however, the two noncritical activities are on the same path ε are not separated by a critical activity. Here, a delay in one of the activities duces the slack time available for the others on the path because activities alc a noncritical path "share" the available slack time. In this case, further investi tive analyses are required to accurately determine the total effect on the en project.

Most projects of any size are solved using software specifically designed project scheduling. When multiple delays are incurred, the easiest way to de mine the effect to the project completion time and the critical path is simpl reenter the updated information and re-solve the model.

5.4 A Linear Programming Approach to PERT/CPM

Given the estimated completion times for each activity and the immediate pı cessor relationships, PERT/CPM networks can actually be modeled and solv linear programs. One way to do this is to define variables X_A, X_B, X_C, and so c represent the start times for each of the corresponding activities. The const set consists of the nonnegativity constraints and one constraint for each imme predecessor relationship in the project. These immediate predecessor const state that:

$$\text{Activity Start Time} \geq \text{Immediate Predecessor Finish Time}$$

Since the finish time for any activity equals its start time plus its completion we can restate the relationship above as:

Linear Programming Constraints for PERT/CPM

Start Time for an Activity ≥
Start Time for the Immediate Predecessor Activity +
Immediate Predecessor's Activity Completion Time

For example, in the Klonepalm 2000 network developed in the last sectio activities C (completion time = 5) and F (completion time = 25) are imn predecessors for activity G. Thus two constraints of this model are:

$$X_G \geq X_C + 5$$
$$X_G \geq X_F + 25$$

Applying this approach to the entire Klonepalm 2000 network, we find that the complete constraint set for this model is:

* add a dummy finish node

* include constraints for its predecessors

$$
\begin{aligned}
X_E &\geq X_D + 20\\
X_H &\geq X_D + 20\\
X_J &\geq X_D + 20\\
X_J &\geq X_I + 30\\
X_D &\geq X_G + 14\\
X_G &\geq X_C + 5\\
X_G &\geq X_F + 25\\
X_C &\geq X_B + 15\\
X_I &\geq X_A + 90\\
X_F &\geq X_A + 90\\
X_B &\geq X_A + 90\\
\text{All } X\text{'s} &\geq 0
\end{aligned}
$$

CALCULATING THE EARLIEST START AND FINISH TIMES

* Then OF is min X_{FIN}. (much more meaningful!)

It can be shown that if we make the objective function: $\cancel{\text{MIN } \Sigma}$X's (in this model, ~~MIN $X_A + X_B + X_C + X_D + X_E + X_F + X_G + X_H + X_I + X_J$~~), although the actual value of the objective function is meaningless, the resulting set of X's will give the *earliest* start (ES) times for each of the activities. The earliest finish (EF) times can then be calculated by adding the activity completion times to these earliest start times. *The overall project completion time is then the maximum of these earliest finish times.*

CALCULATING THE LATEST START AND FINISH TIMES

After solving this linear program and determining that 194 is the expected project completion time, we solve a second linear program. This time we assume that the precedence relations are reversed (the arrows on the activity on the PERT/CPM network now point in the opposite direction), and the problem is solved again with constraints now representing this set of reversed directions. The *latest finish* (LF) times are then found by subtracting the X values generated by this second linear program from 194. Then the *latest start* (LS) times are found by subtracting the activity completion times from the LS times.

5.5 Obtaining Results Using Excel

The Klonepalm model is small enough so that a hand solution is neither tedious nor difficult. To solve this problem using an Excel spreadsheet, one could program each cell's ES and LF times individually as shown in columns D and G of Figure 5.6. The formulas for the cells in columns D and G are given in columns I and J respectively. These formulas show that the ES times must equal the maximum of the corresponding EF times of its immediate predecessor activities listed in column B and the LF times must equal the minimum of the LS for which the corresponding activity is an immediate predecessor. As you can imagine, doing this can take much longer than solving the problem by hand. In fact, you are actually solving the problem by inputting the formulas; all Excel is doing is performing the simple arithmetic operations.

Another approach is to solve the model as a linear program as discussed in the last section. But again, the constraints are problem specific, and entering each constraint into the Solver dialogue box would require about as much work as solving the model by hand.

Hence, if the additional \$250,000 were spent, the net additional expected pı E(NP), would be:

$$E(NP) = \$450{,}976.40 - \$250{,}000.00 = \$200{,}976.40$$

Because this is less than the net expected profit of \$335,751.20 if the \$250,000 i spent, the additional staff training should not be undertaken. Thus manage should not pursue either of the two extra spending options for sales or staff traiı

IS IT APPROPRIATE TO USE THE EXPECTED VALUE APPROACH?

A word of caution must be offered about using the expected value approac. expected value is a "long-run average" value, which means that if the project repeated over and over again, in the long run it would not pay to spend the tional \$250,000 each time for staff training. However, many projects, incl this one, are performed only once. Hence, while the results from an expected analysis can be used as a guide, the decision maker should also consider otheı hunches, and judgments before making a final decision about whether or ı spend the \$250,000 on additional staff training.

5.10 The Critical Path Method (CPM)

The **critical path method (CPM)** is a deterministic approach to project pla based on the assumption that an activity's completion time can be determine certainty. This time depends only on the amount of money allocated to the ity. The process of reducing an activity's completion time by committing tional monetary resources is known as *crashing*.

CPM assumes that there are two crucial time points for each activity: *normal completion time* (T_N), achieved when the usual or *normal cost* (C_N) is s complete the activity; and (2) its *crash completion time* (T_C), the minimum p completion time for the activity; the T_C is attained when a maximum *cr* (T_C) is spent. The assumption is that spending an amount greater than $C_($ activity will not significantly reduce the completion time any further.

To illustrate this CPM assumption, think of a building project that i the construction of a large brick wall. The wall will be completed in a norı if the normal cost is paid to one bricklayer. If extra funds are available to h bricklayers, the completion time should be less; a third bricklayer could rec completion time even more. But there comes a point when the addition of bricklayer will not significantly reduce the time further. Plaster between th takes a certain time to dry, regardless of the number of bricklayers. Carrie extreme, if there are more bricklayers than the number of bricks required wall, the completion time cannot be reduced further. Hence, an activity mum crash cost is the cost at which most of its significant time reduction l achieved. CPM analyses are based on the following **linearity assumption**:

> **CPM Linearity Assumption**
>
> If any amount between C_N and C_C is spent to complete an activity, the *percent decrease* in the activity's completion time from its normal time T_N to its crash ı equals the *percentage increase* in cost from its normal cost to its crash cost.

Figure 5.20 illustrates the linearity concept of crashing. Here:

$R = T_N - T_C$ = the maximum possible time reduction (crashing) of an activity

$E = C_C - C_N$ = the maximum *additional* (crash) costs required to achieve the maximum time reduction

$M = E/R$ = the *marginal* cost of reducing an activity's completion time by one unit

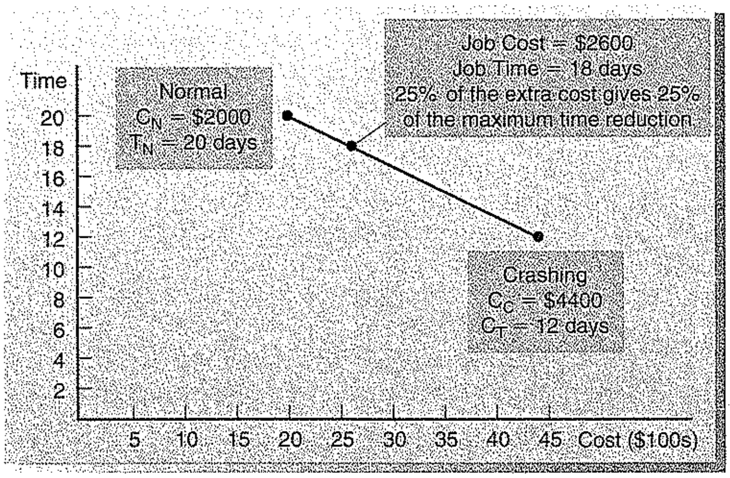


FIGURE 5.20 CPM Linearity Assumption

Figure 5.20 shows that as costs are increased from the normal cost, C_N = \$2000, to the maximum crash cost, C_C = \$4400, the activity's completion time is reduced proportionately from the normal time of T_N = 20 days to the crash time of T_C = 12 days. The maximum time reduction is R = 20 − 12 = 8 days, and the maximum additional cost is E = \$4400 − \$2000 = \$2400. Thus the cost per day reduction is M = E/R = \$2400/8 = \$300/day.

If management allocates \$2600 to complete the activity (\$600 more than its normal cost of \$2000), the time reduction from this increase is \$600/\$300 = 2 days. That is, the activity is now expected to take 20 − 2 = 18 days to complete.

MEETING A DEADLINE AT MINIMUM COST

Suppose management is willing to commit additional monetary resources in an attempt to meet a deadline date, D. It would first check to see whether this can be accomplished by spending the normal costs for the activities. In other words, a deterministic PERT/CPM approach (discussed in Section 7.5.3) can be applied to a network using the normal activity times. If this analysis determined that the project could, indeed, be accomplished by time D, no additional funds would have to be spent on the project.

If, however, the completion time of the project using normal times exceeds the target completion date, management will need to spend additional resources to "crash" some of the activities to meet the target deadline. Its objective is to meet the target date at minimal additional cost. To illustrate this concept, consider the problem faced by management at Baja Burrito Restaurants.

BAJA BURRITO RESTAURANTS

Baja Burrito Restaurants, commonly called BB's, is a chain of fast-food Mexic style restaurants with over 100 locations throughout the Southwest and Gı Plains states. It features such items as the Taco Loco Grande, the Quesadilla Q tro (a quesadilla made with four different cheeses), and specialty burritos nar after many of the states BB's services (such as the Arizona burrito, the Texas b rito, and the Oklahoma burrito). Like most fast-food restaurants, the basic s and décor of the individual restaurants do not vary greatly from restauran restaurant, although the basic design must be modified somewhat to adjust to constraints and to conform to local codes and ordinances.

BB's is planning to open a new restaurant in Lubbock, Texas, near the site w Texas Tech University plays its football games. It wishes to have the restaurant b and operational prior to the first Texas Tech home football game on Septembe which is 19 weeks away. Table 5.12 details the activities and the immediate prede sor relations in the first two columns and the approximate normal times (in we and costs (in $1000s) for building a restaurant in the third and fourth columns. T under normal conditions, it costs BB's about $200,000 to construct a new restaur

Based on the normal time estimates, using the PERT-CPM.xls template can show that it will take 29 weeks for BB's to complete the construction pro Since this does not meet the 19-week deadline, management at BB's has reque department heads and project engineers look for ways to speed up the indiv activities. These are reflected in the last two columns in Table 5.12. Therefo all activities are crashed to their minimum times, the restaurant could be bu 17 weeks at a cost of $300,000.

Management is willing to assume that spending additional funds up t crash amounts submitted by the department heads and engineers will reduc activity completion times proportionately. They are seeking a minimum co tivity schedule for building the new Lubbock Baja Burrito restaurant tha meet the 19-week deadline.

TABLE 5.12 Activity Chart for Baja Burrito Restaurants

Activity	Immediate Predecessors	Normal Time	Normal Cost	Crash Time
A. Plan revisions/approvals	—	5	25	3
B. Grade land	A	1	10	0.5
C. Purchase materials	A	3	18	1.5
D. Order/receive equipment	A	2	8	1
E. Order/receive furniture	A	4	8	1.5
F. Pour concrete floor	B,C	1	12	0.5
G. Erect frame	F	4	20	2.5
H. Install electrical	G	2	12	1.5
I. Install plumbing	G	4	13	2.5
J. Install drywall/roof	H,I	2	10	1.5
K. Construct bathrooms	I	2	8	1
L. Install equipment	D,J	3	14	1.5
M. Finish/paint inside	K,L	3	10	1.5
N. Tile floors	M	3	6	1
O. Install furniture	E,M	4	8	2.5
P. Finish/paint outside	J	4	18	2.5
TOTAL		29*	$200	17*

*As determined by the PERT.xls template.

SOLUTION

Figure 5.21 shows the PERT/CPM network for this model. If this model were solved by hand, the first step would be to determine whether construction of the restaurant would meet the 19-week deadline using the normal time/cost data. As mentioned earlier, using normal times/costs would give a 29-week completion time determined by a critical path consisting of activities A, C, F, G, I, J, L, M, and O.

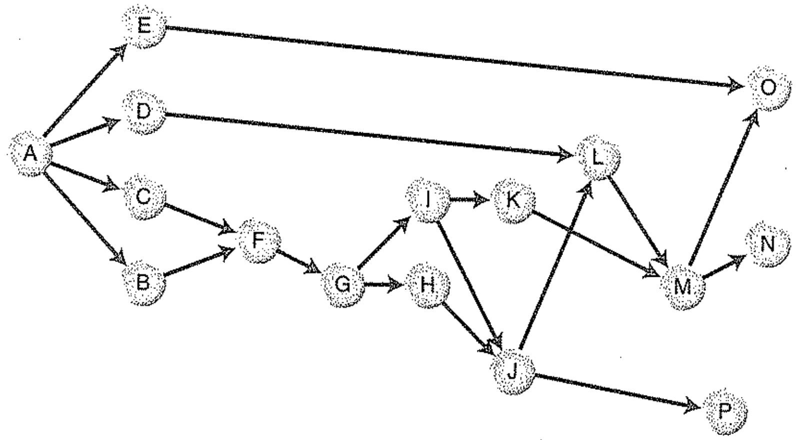


FIGURE 5.21
PERT/CPM Network for Baja Burrito Restaurants

Thus to meet the 19-week deadline, some of the activities must be crashed. Table 5.13 details the maximum time reductions, R ($=T_N - T_C$), the extra costs for these reductions, E, and the marginal cost per week reductions, M (=E/R).

In order to reduce the project time, the completion time of one or more of the critical activities must be crashed. When the completion time for a critical activity is reduced by a large enough amount, however, other paths will also become critical. To achieve further time reductions, activities on *all* critical paths must be crashed.

TABLE 5.13 R, E, and M Values for Baja Burrito

Activity	Maximum Reduction R	Extra Cost E	Cost Per Week Reduction M = E/R
A	2.0	11	5.50
B	0.5	5	10.00
C	1.5	4	2.67
D	1.0	4	4.00
E	2.5	7	2.80
F	0.5	3	6.00
G	1.5	10	6.67
H	0.5	5	10.00
I	1.5	8	5.33
J	0.5	6	12.00
K	1.0	4	4.00
L	1.5	8	5.33
M	1.5	8	5.33
N	2.0	3	1.50
O	1.5	6	4.00
P	1.5	8	5.33

A heuristic approach to determine the amount of time each activity shou crashed can be developed by taking into account the following: (1) the project is reduced only when activities on the critical path are reduced; (2) the maxi time reduction for each activity is limited; and (3) the amount of time an ac on the critical path can be reduced before another path also becomes a critica is limited. For *very* small problems, an approach based on these observatior work rather well, but as the number of critical paths increases, the procedu comes cumbersome rather rapidly.

LINEAR PROGRAMMING APPROACH TO CRASHING

Fortunately, the use of such a heuristic approach is unnecessary. A simple m cation to the linear program given in Section 5.4 is all that is required. Fo model, we now define two variables for each activity, j.

X_j = start time for the activity
Y_j = the amount by which the activity is to be crashed

Since the normal cost must always be paid, the objective is to minimi sum of the *additional* funds spent to reduce the completion times of activitie cost per unit reduction for an activity is M_j, and the amount of time the act reduced is the decision variable, Y_j. Therefore, the total extra amount spent ing the activity is M_jY_j. Because we want to minimize the total additional spent to crash the project, the objective function is the sum of all such costs:

$$\text{MIN} \sum_j M_jY_j$$

For Baja Burrito Restaurants the objective function is:

$$\text{MIN } 5.5Y_A + 10Y_B + 2.67Y_C + 4Y_D + 2.8Y_E + 6Y_F + 6.67Y_G + 10Y\text{:}$$
$$5.33Y_I + 12Y_J + 4Y_K + 5.33Y_L + 5.33Y_M + 1.5Y_N + 4Y_O + 5.3$$

Constraints

There are three types of constraints in this approach:

1. ***No activity can be reduced more than its maximum time reduction.***

For each activity, there is a constraint of the form:

$$Y_j \le R_j$$

2. ***The start time for an activity must be at least as great as the finish all immediate predecessor activities.***

This represents a series of constraints similar to those described in Sectio the form:

$$\text{(Start Time for an Activity)} \ge$$
$$\text{(Finish Time for an Immediate Predecessor of the Activity)}$$

Now, however, since the activity finish times are reduced by the ar time each activity is crashed, these constraints have the form:

$$\text{(Start Time for an Activity)} \ge$$
$$\text{(Start Time for a Predecessor Activity)} +$$
$$\text{(Normal Completion Time of the Predecessor Activity)} -$$
$$\text{(Time the Predecessor Activity is Crashed)}$$

There is one such constraint for each immediate predecessor relationship. (This is equivalent to saying that there is one constraint for each arc in the PERT/CPM network.) In this project, for example, activity I (which has a normal completion time of four weeks) is one of the immediate predecessors for activity J. Thus one of the constraints in the linear programming formulation would be:

$$X_J \geq X_I + (4 - Y_I)$$

3. ***The project must be completed by its deadline, D.***
Since the project completion time is determined by the maximum of the finish times of the terminal activities[3] in the project (the ones that are not predecessors for any other activities), we add constraints of the form:

$$\text{(Finish Time for a Terminal Activity)} \leq D$$

or for each *terminal* activity,

$$\begin{gathered}\text{(Activity Start Time)} + \text{(Activity's Normal Completion Time)} - \\ \text{(Time Activity is Crashed)} \leq D\end{gathered}$$

In this model, activities N, O, and P, having normal completion times of 3, 4, and 4, respectively, are not predecessors for any other activities. Thus the following constraints would be added:

$$\begin{aligned} X_N + 3 - Y_N &\leq 19 \\ X_O + 4 - Y_O &\leq 19 \\ X_P + 4 - Y_P &\leq 19 \end{aligned}$$

The complete linear programming model for Baja Burrito Restaurants is then:

$$\begin{aligned} \text{MIN } & 5.5Y_A + 10Y_B + 2.67Y_C + 4Y_D + 2.8Y_E + 6Y_F + 6.67Y_G + 10Y_H \\ & + 5.33Y_I + 12Y_J + 4Y_K + 5.33Y_L + 5.33Y_M + 1.5Y_N + 4Y_O + 5.33Y_P \\ \text{ST} & \end{aligned}$$

$$\left.\begin{aligned} Y_A &\leq 2.0 \\ Y_B &\leq 0.5 \\ Y_C &\leq 1.5 \\ Y_D &\leq 1.0 \\ Y_E &\leq 2.5 \\ Y_F &\leq 0.5 \\ Y_G &\leq 1.5 \\ Y_H &\leq 0.5 \\ Y_I &\leq 1.5 \\ Y_J &\leq 0.5 \\ Y_K &\leq 1.0 \\ Y_L &\leq 1.5 \\ Y_M &\leq 1.5 \\ Y_N &\leq 2.0 \\ Y_O &\leq 1.5 \\ Y_P &\leq 1.5 \end{aligned}\right\} \quad (1)$$

[3] If you do not wish to identify the terminal activities, you can simply require *all* finish times not to exceed D. This will add several redundant constraints, but for small problems these will not significantly impact the solution time of the linear program.

$$
\left.\begin{aligned}
X_B &\geq X_A + (5 - Y_A)\\
X_C &\geq X_A + (5 - Y_A)\\
X_D &\geq X_A + (5 - Y_A)\\
X_E &\geq X_A + (5 - Y_A)\\
X_F &\geq X_B + (1 - Y_B)\\
X_F &\geq X_C + (3 - Y_C)\\
X_G &\geq X_F + (1 - Y_F)\\
X_H &\geq X_G + (4 - Y_G)\\
X_I &\geq X_G + (4 - Y_G)\\
X_J &\geq X_H + (2 - Y_H)\\
X_J &\geq X_I + (4 - Y_I)\\
X_K &\geq X_I + (4 - Y_I)\\
X_L &\geq X_D + (2 - Y_D)\\
X_L &\geq X_J + (2 - Y_J)\\
X_M &\geq X_K + (2 - Y_K)\\
X_M &\geq X_L + (3 - Y_L)\\
X_N &\geq X_M + (3 - Y_M)\\
X_O &\geq X_E + (4 - Y_E)\\
X_O &\geq X_M + (3 - Y_M)\\
X_P &\geq X_J + (2 - Y_J)
\end{aligned}\right\} \quad (2)
$$

$$
\left.\begin{aligned}
X_N + 3 - Y_N &\leq 19\\
X_O + 4 - Y_O &\leq 19\\
X_P + 4 - Y_P &\leq 19
\end{aligned}\right\} \quad (3)
$$

All X's and Y's ≥ 0

The constraints could be rewritten so that they resemble our usual linear gramming form of having the variables on the left side of the constraints only constants on the right. Regardless, we see that even for this relat small problem, the linear program consists of 32 variables and 39 funct constraints.

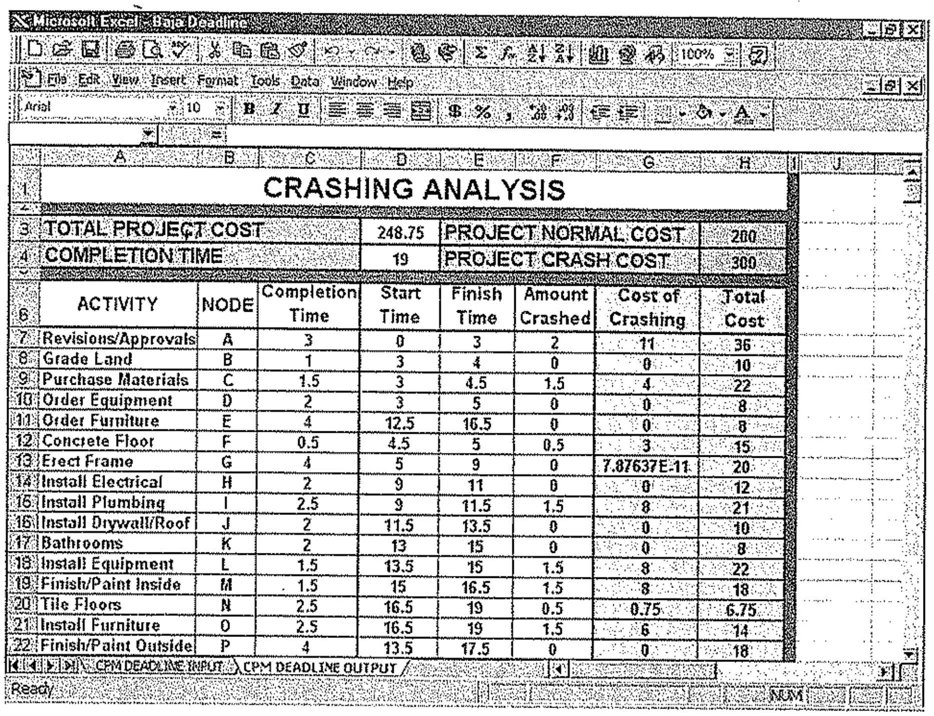
USING CPM-DEADLINE.xls TEMPLATE

Fortunately, one does not actually have to write the linear program for (many computer packages do this automatically. We have included the (Deadline.xls template on the accompanying CD-ROM to do just that. Figure shows the results from the CPM DEADLINE OUTPUT worksheet for the Burrito Restaurant model. We see that the project can be completed in 19 at a cost of $248,750 by crashing activities A, C, F, I, L, M, N, and O by c amounts. A scheduling of the activities that meets this deadline is sho columns C and D. (*Incidentally, the* 7.87637×10^{-11} *entry for the cost of crashi activity G is simply a roundoff error generated internally from solving the linear pr This number is actually 0.*)

OPERATING OPTIMALLY WITHIN A FIXED BUDGET

The CPM approach presented for the Baja Burrito Restaurants model sou find the minimum cost of constructing the restaurant within 19 weeks. Many ects, however, including construction projects, marketing campaigns, a search and development studies, must operate within a given fixed budget. I cases, the objective is to complete the project in minimum time, subject budget restrictions. The CPM approach can be modified for these models.

Baja Deadline.xls



Microsoft Excel - Baja Deadline

CRASHING ANALYSIS

TOTAL PROJECT COST	248.75	PROJECT NORMAL COST	200
COMPLETION TIME	19	PROJECT CRASH COST	300

ACTIVITY	NODE	Completion Time	Start Time	Finish Time	Amount Crashed	Cost of Crashing	Total Cost
Revisions/Approvals	A	3	0	3	2	11	36
Grade Land	B	1	3	4	0	0	10
Purchase Materials	C	1.5	3	4.5	1.5	4	22
Order Equipment	D	2	3	5	0	0	8
Order Furniture	E	4	12.5	16.5	0	0	8
Concrete Floor	F	0.5	4.5	5	0.5	3	15
Erect Frame	G	4	5	9	0	7.87637E-11	20
Install Electrical	H	2	9	11	0	0	12
Install Plumbing	I	2.5	9	11.5	1.5	8	21
Install Drywall/Roof	J	2	11.5	13.5	0	0	10
Bathrooms	K	2	13	15	0	0	8
Install Equipment	L	1.5	13.5	15	1.5	8	22
Finish/Paint Inside	M	1.5	15	16.5	1.5	8	18
Tile Floors	N	2.5	16.5	19	0.5	0.75	6.75
Install Furniture	O	2.5	16.5	19	1.5	6	14
Finish/Paint Outside	P	4	13.5	17.5	0	0	18

CPM DEADLINE INPUT / CPM DEADLINE OUTPUT

FIGURE 5.22
CPM DEADLINE OUTPUT Worksheet for Baja Restaurants

BAJA BURRITO RESTAURANTS (CONTINUED)

Baja Budget.xls

Suppose Baja Burrito has a policy of not funding a project for more than $12\frac{1}{2}\%$ above "normal cost" forecasts. In this case, $12\frac{1}{2}\% = \$25,000$, meaning that the maximum spending limit for the project would be \$225,000. Given this spending limit, management is interested in the earliest it can expect the Lubbock BB's construction project to be completed.

SOLUTION

The problem is basically the same as that for the previous model, with the following exceptions:

1. The constraints, labeled (3), which state that the project must be completed within 19 weeks, are eliminated.
2. A new constraint is added stating that the maximum "extra spending" cannot exceed \$25,000:

$$5.5Y_A + 10Y_B + 2.67Y_C + 4Y_D + 2.8Y_E + 6Y_F + 6.67Y_G + 10Y_H$$
$$+ 5.33Y_I + 12Y_J + 4Y_K + 5.33Y_L + 5.33Y_M + 1.5Y_N + 4Y_O + 5.33Y_P \leq 25$$

3. The objective is changed to Min(Max(EF)).

This objective function, however, is not a linear function. In addition, imposing a constraint similar to MIN ΣX's that we used in Section 5.4, this time does not guarantee an optimal solution.

To convert this to a linear program, we can imagine creating a dummy node called "END," signifying the end of the project. All termination activities, activities that are not predecessors for other activities, are then immediate predecessors of this END node. In this case, activities N, O, and P are not predecessors for any other activities and thus would be predecessors for this END node. The end of the PERT/CPM network for this model is shown in Figure 5.23.

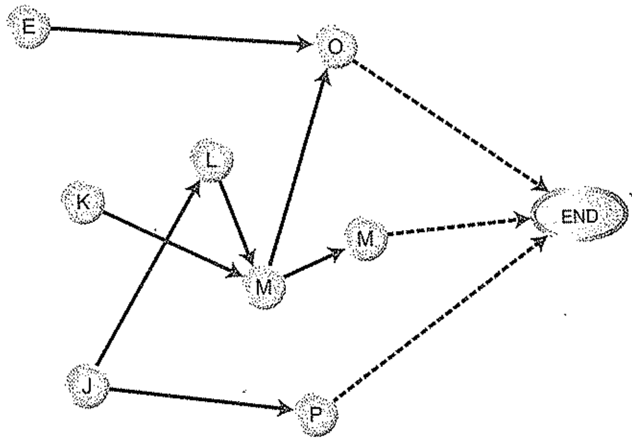


FIGURE 5.23
Termination Nodes (N, O, P) of the Baja Restaurant Network

The constraints of the previous model would then be amended by deleting the constraints (3), adding the constraint above restricting extra spending to at most \$25,000, and adding three more precedence relation constraints:

$$X_{END} \geq X_N + (3 - Y_N)$$
$$X_{END} \geq X_O + (4 - Y_O)$$
$$X_{END} \geq X_P + (4 - Y_P)$$

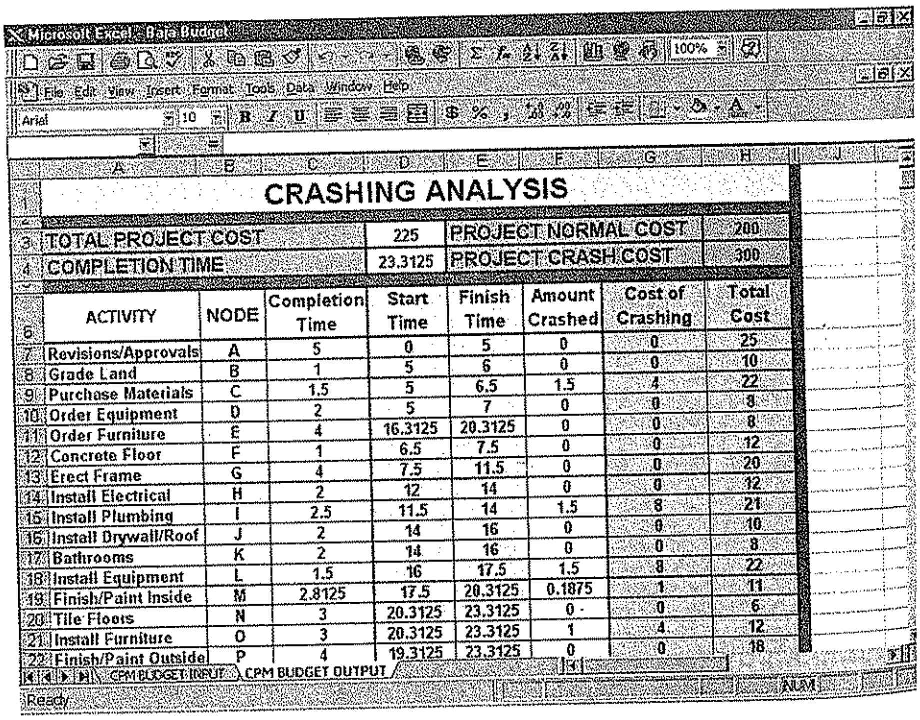
The *linear* objective now is:

$$\text{MIN } X_{END}$$

USING THE CPM-BUDGET.xls TEMPLATE

The above is precisely the approach used in the CPM-Budget.xls template. This template is designed specifically to solve project scheduling models with limited budgets. Input instructions are given in Appendix 5.1. Figure 5.24 shows the CPM BUDGET OUTPUT worksheet for the Baja Burrito model.

Baja Budget.xls



CRASHING ANALYSIS

TOTAL PROJECT COST	225	PROJECT NORMAL COST	200
COMPLETION TIME	23.3125	PROJECT CRASH COST	300

ACTIVITY	NODE	Completion Time	Start Time	Finish Time	Amount Crashed	Cost of Crashing	Total Cost
Revisions/Approvals	A	5	0	5	0	0	25
Grade Land	B	1	5	6	0	0	10
Purchase Materials	C	1.5	5	6.5	1.5	4	22
Order Equipment	D	2	5	7	0	0	8
Order Furniture	E	4	16.3125	20.3125	0	0	8
Concrete Floor	F	1	6.5	7.5	0	0	12
Erect Frame	G	4	7.5	11.5	0	0	20
Install Electrical	H	2	12	14	0	0	12
Install Plumbing	I	2.5	11.5	14	1.5	8	21
Install Drywall/Roof	J	2	14	16	0	0	10
Bathrooms	K	2	14	16	0	0	8
Install Equipment	L	1.5	16	17.5	1.5	8	22
Finish/Paint Inside	M	2.8125	17.5	20.3125	0.1875	1	11
Tile Floors	N	3	20.3125	23.3125	0	0	6
Install Furniture	O	3	20.3125	23.3125	1	4	12
Finish/Paint Outside	P	4	19.3125	23.3125	0	0	18

FIGURE 5.24
CPM BUDGET OUTPUT Worksheet for Baja Burrito Restaurants