Problem 1
Consider the following linear program:

$$\text{max } 3x_1 + 2x_2$$
$$\text{s.t.}$$
$$2x_1 + x_2 \leq 8$$
$$x_1 + 2x_2 \leq 6$$
$$x_1, x_2 \geq 0$$

a. Graph the feasible region, making sure to label your axes, constraints, and all corner points.
b. Fill in the following table.

<table>
<thead>
<tr>
<th>Extreme Point</th>
<th>O.F. Value</th>
</tr>
</thead>
</table>

c. What is an optimal solution for this LP?
Problem 2
A cargo plane has three compartments for storing cargo: front, center, and back. These compartments have capacity limits on both weight and space as summarized below:

<table>
<thead>
<tr>
<th>Compartment</th>
<th>Weight Capacity (Tons)</th>
<th>Space Capacity (ft³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front</td>
<td>12</td>
<td>7000</td>
</tr>
<tr>
<td>Center</td>
<td>18</td>
<td>9000</td>
</tr>
<tr>
<td>Back</td>
<td>10</td>
<td>5000</td>
</tr>
</tbody>
</table>

Furthermore, the weight of the cargo in the respective compartments must be the same proportion of that compartment’s weight to maintain the balance of the airplane.

The following four cargoes have been offered for shipment on an upcoming flight:

<table>
<thead>
<tr>
<th>Cargo</th>
<th>Weight (Tons)</th>
<th>Volume (ft³/Ton)</th>
<th>Profit ($/Ton)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>500</td>
<td>320</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>700</td>
<td>400</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>600</td>
<td>360</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
<td>400</td>
<td>290</td>
</tr>
</tbody>
</table>

Any portion of these cargoes can be accepted. The objective is to determine how much (if any) of each cargo should be accepted and how to distribute each among the compartments to maximize the total profit for the flight.

Formulate this problem as a linear program to maximize profit.
Problem 3
Consider the scenario presented in Problem 2.

a. Create a generic model.

b. Does the model satisfy the four assumptions of linear programming? Explain your answer carefully.
Problem 4
Let $A$ be an $m \times n$ matrix, $\mathbf{x} \in \mathbb{R}^n$, and $\mathbf{b} \in \mathbb{R}^m$. Carefully explain how you could use the simplex method to determine whether or not the system

$$Ax = b$$

has a solution.
Problem 5
Solve the following LP using the simplex method.

\[
\begin{align*}
\text{max} & \quad 4x_1 + 2x_2 \\
\text{s.t.} & \quad x_1 + x_2 \leq 10 \\
& \quad x_1 \geq 5 \\
& \quad x_1, \quad x_2 \geq 0
\end{align*}
\]
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