Practice Final
MAT-16C Short Calculus - III

Spring 2011

Name _____

- **This test is closed notes, closed book.**
- **Laptops and calcultors are NOT allowed.**
- **There are 4 pages and 11 questions total.**
- **You can leave an answer as a numerical expression without computing the final value. For example, this is a perfectly acceptable answer :**

  $((250-63)/(1-e^{(-6*3.5)})) * ln(27/168)$. **Show your work clearly !!.**
- **The maximum score in the test is 150 points.**

Signature _____

Problem	Score	Max Possible
1		10
2		15
3		6
4		9
5		18
6		12
7		10
8		24
9		20
10		15
11		11
Total		150

1. Let $f(x,y) = cos(1+xy^2)$.
   (a) **(4 pts)** Compute f_y.
   (b) **(6 pts)** Compute f_{yx}.
2. Consider the function $f(x,y) = 5 + x^2 - x^2y - y^2 - \frac{1}{3}y^3$.
   (a) **(8 pts)** Find all critical points of $f(x,y)$.
   (b) **(7 pts)** Decide whether each critical point found in (a) is a relative minimum, relative maximum, saddle point or indeterminable.
3. Write the n-th term in the following sequences :
   (a) **(2 pts)** $-\frac{1}{2}, \frac{4}{3}, -\frac{9}{4}, \frac{16}{5}, \ldots$
   (b) **(2 pts)** $2, 2, \frac{8}{6}, \frac{16}{24}, \frac{32}{120}, \ldots$
   (c) **(2 pts)** $1, \frac{5}{8}, \frac{7}{15}, \frac{9}{24}, \frac{11}{35}, \frac{13}{48}, \ldots$
4. Determine if the following sequences converge or diverge. If the sequence converges, write the limit.
   (a) **(3 pts)** $a_n = \frac{2^n+1}{3 \cdot 2^n}$
   (b) **(3 pts)** $a_n = (-1)^n(\frac{1}{n^2+3})$
   (c) **(3 pts)** $a_n = (-1)^n(\frac{n+1}{n})$
5. Determine if the following series converge or diverge. Clearly explain why.
   (a) **(6 pts)** $\sum_{n=0}^{\infty} \frac{n}{500n+79}$
   (b) **(6 pts)** $\sum_{n=1}^{\infty} (2n)!(\frac{2}{3})^n$
   (c) **(6 pts)** $\sum_{n=1}^{\infty} \frac{1}{n^2\sqrt{n}}$
6. **(12 pts)** Find the radius and interval of convergence for the power series
$$\sum_{n=1}^{\infty} \frac{3^{-n}}{n+1}(x+1)^n.$$
7. **(10 pts)** Find the sum $\sum_{n=0}^{\infty} \frac{1}{3^n 2^{n-2}}$.
8. Evaluate the following double integrals. (Remember that sometimes it helps to change the order of integration).
   (a) **(12 pts)** $\int_0^2 \int_0^{\sqrt{x}} y(x-y^2)^3 dydx$.
   (b) **(12 pts)** $\int_0^4 \int_{\sqrt{x}}^2 xsin(1+y^5) dydx$.

9. Solve the following differential equations.

   (a) **(10 pts)** $e^{x^2-x}y' + y = 2xy$.

   (b) **(10 pts)** $xy' - 2y = x^2 ln(x)$ with the initial conditions that $y = 0$ when $x = 1$.

10. **(15 pts)** Approximate the definite integral $\int_0^1 xe^{-x^3}dx$ using a 7th-degree Taylor polynomial for the function xe^{-x^3}.

11. **(11 pts)** Minimize the function $f(x, y, z) = x^2 + 2y^2 + 3z^2$ subject to the constraint $3x - 2y + z = \frac{34}{6}$. Find x, y, z at the minimum and the minimum value of the function.