1. Differentiate the following functions. DO NOT SIMPLIFY ANSWERS.

(a) **(4 pts)**

$$y = 3x^2 + x^{-7} - \frac{5}{11}$$

$$\frac{dy}{dx} = \boxed{6x - 7x^{-8}}$$

(b) **(5 pts)**

$$y = x^{100} sin(x)$$

$$\frac{dy}{dx} = 100x^{99} \sin(x) + x^{100}(-\cos x)$$

$$= \boxed{100x^{99} \sin(x) - x^{100} \cos(x)}$$

(c) **(5 pts)**

$$y = \frac{x-2}{3-x^5}$$

$$\frac{dy}{dx} = \frac{(1)(3-x^5) - (x-2)(-5x^4)}{(3-x^5)^2}$$

$$= \frac{3-x^5+5x^5-10x^4}{(3-x^5)^2}$$

$$= \frac{4x^5-10x^4+3}{(3-x^5)^2}$$

all answers are OK.

(d) **(5 pts)**

$$y = (\tan(x))^3$$

$$\frac{dy}{dx} = \boxed{3(\tan(x))^2 \sec^2(x)}$$

Using the chain Rule $f(x) = x^3$, $g(x) = \tan(x)$.

or using $u = \tan(x)$

(e) **(6 pts)**

$$y = (1 + (\cos(x))^2)^3$$

$$\frac{dy}{dx} = \boxed{3\left(1+(\cos(x))^2\right)^2 \, 2\cos(x)(-\sin(x))}$$

Using two applications of the Chain Rule:

1st stage: $f(x) = x^3$, $g(x) = 1+(\cos(x))^2$

2nd stage: $f(x) = x^2$, $g(x) = \cos(x)$

OR

1st stage: $u = 1+(\cos(x))^2$

2nd stage: $u = \cos(x)$.

2. **(10 pts)** Solve the equation for all values of θ in the interval $[0, 2\pi]$:

$$\cos(2\theta) + \sin(\theta) = 0.$$

(Hint: Use a double angle formula for $\cos(2\theta)$ to express the equation as a quadratic equation in $\sin(\theta)$.)

$\cos(2\theta) = 1 - 2\sin^2\theta$.

so $\cos(2\theta) + \sin\theta = 0$

$\Rightarrow 1 - 2\sin^2\theta + \sin\theta = 0$

$\Rightarrow 2\sin^2\theta - \sin\theta - 1 = 0$.

$\Rightarrow (2\sin\theta + 1)(\sin\theta - 1) = 0$

$\Rightarrow \sin\theta = 1$ or $\sin\theta = -\frac{1}{2}$

$\sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$ $\quad \sin\theta = -\frac{1}{2} \Rightarrow \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

so $\theta = \boxed{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

3. **(10 pts)** Find the equation of the tangent line to the graph of the equation

$$y^3 + xy = 8$$

at the point $(0, 2)$.

$$\frac{d}{dx}\left[y^3 + xy\right] = \frac{d}{dx}[8]$$

$$\Rightarrow 3y^2\frac{dy}{dx} + y + x\frac{dy}{dx} = 0$$

$$\Rightarrow (3y^2 + x)\frac{dy}{dx} = -y \Rightarrow \frac{dy}{dx} = \frac{-y}{3y^2 + x}$$

At $(0,2)$, $\frac{dy}{dx} = \frac{-2}{3(2)^2 + 0} = -\frac{1}{6}$

$\therefore$ tangent line : $y - 2 = -\frac{1}{6}(x - 0) \Rightarrow \boxed{y = \frac{-x}{6} + 2}$

4. **(8 pts)** Find the equation of the line *perpendicular* to the tangent line at (0, 1) on the graph of

$$f(x) = \frac{2}{3}x^{3/2} - x + 1.$$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{2}{3}x^{3/2} - x + 1\right] = x^{1/2} - 1$$

At $(0,1)$, $\frac{dy}{dx} = 0^{1/2} - 1 = -1$

So the perpendicular line has slope $= 1$.

$\therefore$ $y - 1 = (1)(x - 0) \Rightarrow \boxed{y = x + 1}$

5. **(7 pts)** Find $f''(x)$ when $f(x) = x\sin(2x)$.

$$f'(x) = (1)\sin(2x) + x\cos(2x)(2)$$

$$= \sin(2x) + 2x\cos(2x).$$

$$f''(x) = \cos(2x)(2) + 2\left[1\cos(2x) + x(-\sin(2x))(2)\right]$$

$$= 2\cos(2x) + 2\cos(2x) - 4x\sin(2x)$$

$$= \boxed{4\cos(2x) - 4x\sin(2x).}$$

6. **(10 pts)** The volume of a cube is increasing at the rate of 3 *in.*^{3}/sec. What is the rate of change of its surface area when each edge is 2 inches long ?

Volume V and length of edge l are related by $V = l^3$

Surface area $S = 6l^2$, $\frac{dV}{dt} = 3$

$$\frac{dV}{dt} = \frac{d[l^3]}{dt} = 3l^2\frac{dl}{dt}$$

~~When $l = 2$~~, $3 = 3(2)^2\frac{dl}{dt} \Rightarrow \frac{dl}{dt} = \frac{1}{4}$

$$\frac{dS}{dt} = \frac{d(6l^2)}{dt} = 12l\frac{dl}{dt} = 12(2)\left(\frac{1}{4}\right)$$

$$= \boxed{6 \text{ inches}^2/\text{sec.}}$$

7. **(5 pts)** Find $\frac{dy}{dx}$ at the point (16, 25) for the equation

$$x^{1/2} + y^{1/2} = 9.$$

$$\frac{d}{dx}\left[x^{1/2} + y^{1/2}\right] = \frac{d}{dx}[9]$$

$$\Rightarrow \frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}\frac{dy}{dx} = 0.$$

$$\Rightarrow \cancel{\frac{1}{2}}y^{-1/2}\frac{dy}{dx} = -\cancel{\frac{1}{2}}x^{-1/2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x^{-1/2}}{y^{-1/2}} = -\frac{y^{1/2}}{x^{1/2}}$$

At (16, 25)

$$\frac{dy}{dx} = -\frac{\sqrt{25}}{\sqrt{16}} = \boxed{-\frac{5}{4}}$$