# Practice Midterm
# MAT-16C Short Calculus - III

## Spring 2011

**Name** _____

- **This test is closed notes, closed book.**
- **Laptops and calcultors are NOT allowed.**
- **There are 8 pages and 7 questions total.**
- **You can leave an answer as a numerical expression without computing the final value. For example, this is a perfectly acceptable answer :**

  $((250-63)/(1-e^{(-6*3.5)}))*ln(27/168)$. **Show your work clearly !!.**
- **The maximum score in the test is 80 points.**

**Signature** _____

| Problem | Score | Max Possible |
|---|---|---|
| 1 | | 10 |
| 2 | | 20 |
| 3 | | 15 |
| 4 | | 10 |
| 5 | | 15 |
| 6 | | 15 |
| 7 | | 6 |
| 8 | | 9 |
| Total | | 100 |

1. Let $f(x, y) = x^2 - y^2$.

   (a) **(8 pts)** Compute $f_{xx}$ and $f_{yy}$.

   (b) **(2 pts)** Show that $f_{xx} + f_{yy}$ is always 0.

2. Consider the function $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$.

   (a) **(12 pts)** Find all critical points of $f(x, y)$. (Hint : There are four)

(b) **(8 pts)** Decide whether each critical point found in (a) is a relative minimum, relative maximum, saddle point or indeterminable.

3. **(15 pts)** The airline industry puts a limit on the size of the checked baggage on any flight. The restriction states that the SUM of the length, breadth and height of the suitcase can be at most 300 inches. Find the dimensions of the suitcase satisfying this restriction that has the MAXIMUM volume. Show that it is in fact a cube.

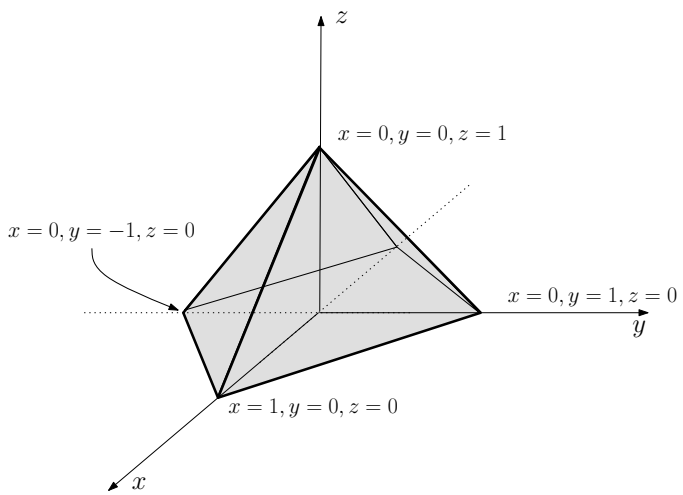
4. **(10 pts)** Compute $\iint_R xy\,dA$ where $R$ is the square region given by $0 \leq x \leq 1$ and $0 \leq y \leq 1$.

5. **(15 pts)** Compute $\int_0^{\ln 10}\int_{e^x}^{10} \frac{1}{\ln y} dydx$. (Hint : You may consider changing the order of integration)

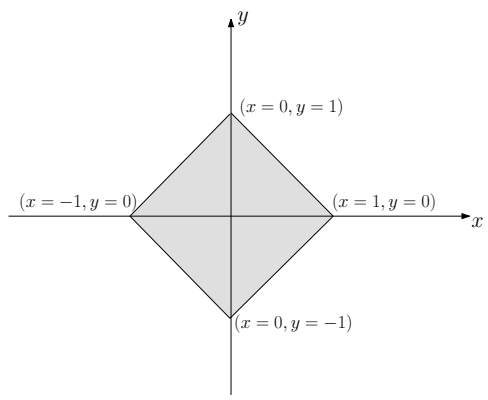
6. You will compute the volume of a pyramid using double integrals. Consider the pyramid whose base is on the $xy$ plane and the four slanted sides are given by the following four planes :

$$\begin{aligned} z &= 1 - x - y \\ z &= 1 - x + y \\ z &= 1 + x - y \\ z &= 1 + x + y \end{aligned}$$

Note that this means the top vertex of the pyramid is the point $(x = 0, y = 0, z = 1)$; all the four planes meet at that point. Figure 1(a) shows a 3D view of the pyramid and Figure 1(b) shows the base drawn on the $xy$ plane.




(a) A 3D view of the pyramid




(b) A drawing of the base of the pyramid

Figure 1: Diagrams for Problem 6

(a) **(8 pts)** Determine the equations of the four lines forming the sides of the base in the $xy$ plane.

(b) **(2 pts)** Observe that the pyramid is symmetrical about the $z$ axis and its volume is four times the volume of the section of the pyramid lying in the first octant. (Recall that the first octant is the region in space satisfying $x \geq 0, y \geq 0, z \geq 0$). This section of the pyramid is the part of the first octant bounded by one of the four planes given above. Write the equation of this plane.

(c) **(5 pts)** Use the observations made in (b) to set up (but DO NOT EVALUATE) the double integral to compute the volume of the pyramid.

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7. Write the $n$-th term in the following sequences :

   (a) **(2 pts)** $-\frac{1}{2}, \frac{4}{3}, -\frac{9}{4}, \frac{16}{5}, \ldots$

   (b) **(2 pts)** $2, 2, \frac{8}{6}, \frac{16}{24}, \frac{32}{120}, \ldots$

   (c) **(2 pts)** $1, \frac{5}{8}, \frac{7}{15}, \frac{9}{24}, \frac{11}{35}, \frac{13}{48}, \ldots$

8. Determine if the following sequences converge or diverge. If the sequence converges, write the limit.

   (a) **(3 pts)** $a_n = \frac{2^n+1}{3 \cdot 2^n}$

   (b) **(3 pts)** $a_n = (-1)^n(\frac{1}{n^2+3})$

   (c) **(3 pts)** $a_n = (-1)^n(\frac{n+1}{n})$