1. Find the following limits.

(a) **(5 pts)** (Hint : Factorize and divide out)

$$\lim_{x \to 3} \frac{x^2 - 9}{x^2 - 5x + 6}.$$

$$= \lim_{x \to 3} \frac{(x+3)\cancel{(x-3)}}{(x-2)\cancel{(x-3)}} = \lim_{x \to 3} \frac{x+3}{x-2}$$

$$= \frac{3+3}{3-2} = 6$$

(b) **(5 pts)** (Hint : Rationalize the numerator)

$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}.$$

$$= \lim_{x \to 4} \frac{\sqrt{x}-2}{x-4} \frac{(\sqrt{x}+2)}{(\sqrt{x}+2)} = \lim_{x \to 4} \frac{\cancel{(x-4)}}{\cancel{(x-4)}(\sqrt{x}+2)}$$

$$= \frac{1}{\sqrt{4}+2} = \frac{1}{4}$$

(c) **(6 pts)** (Hint : Add fractions first)

$$\lim_{x\to 0} \frac{\frac{1}{x+1}+\frac{1}{x-1}}{x}.$$

$$= \lim_{x\to 0} \frac{\left(\frac{(x-1)+(x+1)}{(x+1)(x-1)}\right)}{x} = \lim_{x\to 0} \frac{\left(\frac{2x}{x^2-1}\right)}{x}$$

$$= \lim_{x\to 0} \frac{2\cancel{x}}{\cancel{x}(x^2-1)}$$

$$= \lim_{x\to 0} \frac{2}{x^2-1}$$

$$= \frac{2}{-1} = -2$$

(d) **(4 pts)**

$$\lim_{x\to\infty} \frac{2x^2+3x}{500+3x^2}.$$

Using the rule for rational functions:
Degree of numerator = degree of denominator.

$$\therefore \lim_{x\to\infty} \frac{2x^2+3x}{3x^2+500} = \frac{2}{3}$$

2. **(3 pts)** Let $f(x) = x + 2$ and $g(x) = x^2 - 5x$. Write the expression for $g(f(x))$.

$$g(f(x)) = (x+2)^2 - 5(x+2) = x^2 + 4 + 4x - 5x - 10$$
$$= \boxed{x^2 - x - 6}$$

3. **(4 pts)** Let $h(x) = \frac{1}{(x+3)^3}$ and $f(x) = x + 3$. What should $g(x)$ be such that $h(x) = g(f(x))$?

$$g(x) = \frac{1}{x^3}$$

4. **(8 pts)** Find the inverse of the function : $f(x) = 7 - \sqrt{x-3}$.

$$y = 7 - \sqrt{x-3}$$

switch x + y :

$$x = 7 - \sqrt{y-3}$$

Solve for y :

$$\sqrt{y-3} = 7 - x$$
$$\Rightarrow (\sqrt{y-3})^2 = (7-x)^2$$
$$\Rightarrow y - 3 = (7-x)^2$$
$$\Rightarrow y = 3 + (7-x)^2$$

$\boxed{\text{So inverse is } 3+(7-x)^2}$

5. **(5 pts)** Determine if the following function is continuous at $x = 1$.

$$f(x) = \begin{cases} x^2 + 3x + 1 & x < 1 \\ 3 & x = 1 \\ \sqrt{x+8} & x > 1 \end{cases}$$

Show all your work and justify your answer for full credit.

1. $f(1)$ is defined, $f(1) = 3$.

2. Check if limit exists:

$$\lim_{x \to 1^-} f(x) = 1^2 + 3(1) + 1 = 5.$$

$$\lim_{x \to 1^+} f(x) = \sqrt{1+8} = 3.$$

Since left limit $\neq$ right limit, the limit does not exist and so the function is not continuous.

6. **(5 pts)** Find all points of intersection of the curves defined by the two equations : $y = x^2 + x$ and $y = 2x^2 - 2x + 2$.

Set $x^2 + x = 2x^2 - 2x + 2$.

$\Rightarrow 2x^2 - x^2 - 2x - x + 2 = 0$

$\Rightarrow x^2 - 3x + 2 = 0$

$\Rightarrow (x-2)(x-1) = 0$

$\Rightarrow x = 2, \quad x = 1$

Using $x = 2$, $y = 2^2 + 2 = 6$

Using $x = 1$, $y = 1^2 + 1 = 2$

So pts. of intersection $(2, 6)$ and $(1, 2)$.

7. **(5 pts)** Find the equation of the circle whose diameter has end-points given by $(-1, 3)$ and $(3, 1)$.

Center is midpoint of endpoints of diameter:

$$\left(\frac{-1+3}{2}, \frac{3+1}{2}\right) = (1, 2).$$

$$\text{Radius} = \frac{1}{2} \times \text{diameter} = \frac{1}{2}\sqrt{(3-(-1))^2 + (1-3)^2}$$

$$= \frac{1}{2}\sqrt{16+4}$$

$$= \frac{1}{2}\sqrt{20}$$

$\therefore$ equation:

$$\boxed{(x-1)^2 + (y-2)^2 = \left(\frac{\sqrt{20}}{2}\right)^2}$$

8. **(10 pts)** Use the definition of the derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

to find the derivative of $f(x) = 2x^2 - 3x + 7$.

$$f'(x) = \lim_{h \to 0} \frac{\left(2(x+h)^2 - 3(x+h) + 7\right) - \left(2x^2 - 3x + 7\right)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{2x^2} + 2h^2 + 4xh \cancel{-3x} - 3h \cancel{+7} \cancel{-2x^2} \cancel{+3x} \cancel{-7}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2h^2 + 4xh - 3h}{h} =$$

$$f'(x) = \lim_{h \to 0} \frac{2h^{\cancel{2}}}{\cancel{h}} + \frac{4x\cancel{h}}{\cancel{h}} - \frac{3\cancel{h}}{\cancel{h}}$$

$$f'(x) = \boxed{4x - 3}$$

9. **(10 pts)** Find the horizontal and vertical asymptotes for the function $f(x) = \frac{x^2+x-6}{(x+1)(x-2)}$.

For horizonal asymptote, we need

$$\lim_{x\to\infty} \frac{x^2+x-6}{(x+1)(x-2)} = \lim_{x\to\infty} \frac{x^2+x-6}{x^2-x-2}$$

$$= 1/1 = 1$$

(using the rule for rational functions).

~~Also~~ Also $\lim_{x\to-\infty} \frac{x^2+x-6}{x^2-x-2} = \frac{1}{1} = 1.$

So $\boxed{y=1 \text{ is the horizontal asymptote.}}$

For vertical asymptote, check where denominator is 0.

$$(x+1)(x-2) = 0$$

$$\Rightarrow x=-1, \quad x=2.$$

At $x=-1$ we get $\frac{(-1)^2+(-1)-6}{(-1+1)(-1-2)}$

$$= \frac{-5}{0}$$

At $x=2$ we get $\frac{2^2+2-6}{(2+1)(2-2)} = \frac{0}{0}$

So only $\boxed{x=-1 \text{ is a vertical asymptote}}$