1. Differentiate the following functions. DO NOT SIMPLIFY ANSWERS.

(a) **(4 pts)**

$$y = x^5 + \frac{4}{x^2} - 10^2$$

2 2

$$\boxed{\frac{dy}{dx} = 5x^4 - \frac{(2)(4)}{x^3}}$$

$$= \boxed{5x^4 - \frac{8}{x^3}}$$

both are OK!

(b) **(5 pts)**

$$y = x^{-5} sin(x)$$

3 2

$$\boxed{\frac{dy}{dx} = -5x^{-6}\sin(x) + x^{-5}\cos(x)}$$

(c) **(4 pts)**

$$y = \frac{x-2}{3-x^5}$$

$$\frac{dy}{dx} = \boxed{\frac{(1)(3-x^5) - (x-2)(-5x^4)}{(3-x^5)^2}}$$

$$= \boxed{\frac{4x^5 - 10x^4 + 3}{(3-x^5)^2}}$$

both are OK.

(d) **(6 pts)**

$$y = (\tan(2x+4))^3$$

$$\frac{dy}{dx} = \boxed{3(\tan(2x+4))^2 (\sec^2(2x+4))(2)}$$

$$= \boxed{6\tan^2(2x+4)\sec^2(2x+4)}$$

both are OK.

(e) **(6 pts)**

$$y = \left(\frac{x+2}{x+1}\right)^{-3}$$

Using the chain Rule:

$$\frac{dy}{dx} = \boxed{-3\left(\frac{x+2}{x+1}\right)^{-4}\left(\frac{(1)(x+1)-(1)(x+2)}{(x+1)^2}\right)}$$

$$= \boxed{-3\left(\frac{x+2}{x+1}\right)^{-4}\frac{-1}{(x+1)^2}}$$

both are OK.

2. **(10 pts)** Solve the equation for all values of θ in the interval $[0, 2\pi]$:

$$\cos(2\theta) - \cos(\theta) = 0.$$

(Hint: Use a double angle formula for $\cos(2\theta)$ to express the equation as a quadratic equation in $\cos(\theta)$.)

$$\cos(2\theta) = 2\cos^2\theta - 1$$

so we get

$$2\cos^2\theta - 1 - \cos\theta = 0$$

Factoring: $(2\cos\theta + 1)(\cos\theta - 1) = 0$

$\Rightarrow \cos\theta = -\frac{1}{2}$ or $\cos\theta = 1$

$\Rightarrow \boxed{\theta = \frac{2\pi}{3}, \frac{4\pi}{3}}$ or $\boxed{\theta = 0, \cancel{2\pi}}$

3. **(10 pts)** Find the equation of the tangent line to the graph of the equation

$$2xy^3 + 5xy = 14$$

at the point $(2, 1)$.

Using implicit differentiation: $\dfrac{d}{dx}\left[2xy^3 + 5xy\right] = \dfrac{d}{dx}\left[14\right].$

-1 if wrong ans.

4 $\Rightarrow \left[(2x)\left(\cancel{2}\,3y^2 \dfrac{dy}{dx}\right) + (2)y^3\right] + 5x\dfrac{dy}{dx} + 5y = 0$

if wrong, ~~keep~~ check

$\Rightarrow \left(6xy^2 + 5x\right)\dfrac{dy}{dx} = -5y - 2y^3$

2 $\Rightarrow \dfrac{dy}{dx} = \dfrac{-5y - 2y^3}{6xy^2 + 5x}$

At $(2,1)$ we get $\dfrac{dy}{dx} = \dfrac{-5-2}{6(2) + 5(2)} = \dfrac{-7}{\cancel{18}\,22}$ 2

eq. of tangent line:

$\boxed{y - 1 = \dfrac{-7}{22\cancel{8}}(x-2)}$ 2

$1 + \dfrac{14}{22} = \dfrac{36}{22}$

4. **(8 pts)** Find the equation of the line *perpendicular* to the tangent line at the point with $x = 1$ on the graph of the function

$$f(x) = x^2 - 3x.$$

+2

4

$f'(x) = 2x - 3$

At $x = 1$ $f'(1) = 2(1) - 3 = -1$

Slope of the perpendicular $= +2\ \dfrac{-1}{-1} = 1$

4

y coordinate of pt $= +2\ 1^2 - 3(1) = -2.$

$\therefore$ Eq of perp. line: $+2\ y - (-2) = 1(x-1)$

$\Rightarrow \boxed{y + 2 = x - 1}$

$y = x - 3$

5. **(7 pts)** Find $f''(x)$ when $f(x) = x\sin(2x)$. 2

$$f'(x) = (1)\sin(2x) + x\cos(2x)(2)$$ 2

$$= \sin(2x) + 2x\cos(2x)$$

$$f''(x) = \cos(2x)(2) + 2\left[\cos(2x) + x(-\sin(2x)(2))\right]$$ 1 2

$$= \boxed{2\cos(2x) + 2\cos(2x) - 4x\sin(2x)}$$

$$= \boxed{4\cos(2x) - 4x\sin(2x)}$$

all three are OK!

6. **(10 pts)** The volume V of a right circular cone is increasing at 3 $in.^3/sec$. When the radius $r = 1$ inch and height $h = 3$ inches, the rate of change of the radius is $\frac{dr}{dt} = 2$ inches/sec. What is the rate of change of the height $\frac{dh}{dt}$ at this time ? The relation between the volume V, radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$.

Related Rates:
$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{1}{3}\pi r^2 h\right]$$ +3.

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$$\Rightarrow \frac{dV}{dt} = \frac{1}{3}\pi\left[2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\right]$$

5

$$r = 1,\quad h = 3,\quad \frac{dV}{dt} = 3,\quad \frac{dr}{dt} = 2.$$

$$\therefore\ 3 = \frac{1}{3}\pi\left(2(1)(2)(3) + (1)^2\frac{dh}{dt}\right)$$

$$\Rightarrow \boxed{\frac{dh}{dt} = \frac{9}{\pi} - 12}$$

$r^2 = x^2 + y^2$

$2r\frac{dr}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

7. **(10 pts)** Suppose that a point is moving in the plane such that when the point has coordinates $(1,3)$ the rate of change of its x-coordinate is $\frac{dx}{dt} = 2$ and the rate of change of its y-coordinate is $\frac{dy}{dt} = -1$. Find the rate of change of the distance r of this moving point from the origin, when it has coordinates $(1,3)$.

(Hint : Write down the relation between the distance r from the origin and the x and y coordinates of the point. Then use the Related Rates technique to get a relation between the rate of change of the distance $\frac{dr}{dt}$, and $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

7

3 $r = \sqrt{x^2 + y^2}$

$\Rightarrow$ 4 $\frac{dr}{dt} = \frac{1}{2}(x^2+y^2)^{-1/2}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right)$

at $x = 1$, $y = 3$, $\frac{dx}{dt} = 2$, $\frac{dy}{dt} = -1$

3

$\therefore \frac{dr}{dt} = \frac{1}{2}(1^2+3^2)^{-1/2}\left(2(1)(2) + 2(3)(-1)\right)$

$= \frac{1}{2}10^{-1/2}(4-6) = \boxed{-10^{-1/2}}$

$= \boxed{-\frac{1}{\sqrt{10}}}$

both are OK!