1. Compute the following integrals (note that some of them are indefinite integrals and some are definite integrals).

(a) **(4 pts)** $\int \frac{e^{(1+\sqrt{x})}}{2\sqrt{x}} dx$.

Substitute $u = 1+\sqrt{x}$ $du = \frac{1}{2\sqrt{x}} dx$

$$\therefore \int \frac{e^{(1+\sqrt{x})}}{2\sqrt{x}} dx = \int e^u du = e^u$$

$$= \boxed{e^{(1+\sqrt{x})} + C}$$

(b) **(4 pts)** $\int_0^1 x\sqrt{4x+3}dx$.

Substitute $u = 4x+3$

$du = 4dx$

$x = \frac{u-3}{4}$

Limits $x=0$

$\rightarrow u = 4\cdot 0+3 = 3$

$x = 1$

$\rightarrow u = 4\cdot 1+3 = 7$

$$\therefore \int_0^1 x\sqrt{4x+3}\, dx = \int_3^7 \left(\frac{u-3}{4}\right)\sqrt{u}\,\frac{du}{4} = \frac{1}{16}\left(\frac{u^{5/2}}{(5/2)} - \frac{3u^{3/2}}{(3/2)}\right)\Bigg|_3^7$$

$$\boxed{= \frac{1}{16}\left[\left(\frac{7^{5/2}}{(5/2)} - \frac{3(7^{3/2})}{(3/2)}\right) - \left(\frac{3^{5/2}}{5/2} - \frac{3(3)^{3/2}}{3/2}\right)\right]}$$

(c) **(4 pts)** $\int \frac{3x^2-7x-2}{x^3-x}dx$.

$$\frac{3x^2-7x-2}{x^3-x} = \frac{3x^2-7x-2}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

Plug in $x = 1, -1, 0$ in

$$3x^2-7x-2 = A(x-1)(x+1) + B(x+1)x + Cx(x-1)$$

$x=1 \Rightarrow -6 = 2B \Rightarrow B = -3$

$x=-1 \Rightarrow 8 = +2C \Rightarrow C = 4$

$x = 0 \Rightarrow -2 = -A \Rightarrow A = 2$

$$\therefore \frac{3x^2-7x-2}{x^3-x} = \frac{2}{x} + \frac{-3}{x-1} + \frac{4}{x+1}$$

$$\therefore \int \frac{3x^2-7x-2}{x^3-x} = \boxed{2\ln|x| - 3\ln|x-1| + 4\ln|x+1| + C}$$

(d) **(4 pts)** $\int x^2e^x dx$.

Integration by parts :

$$\int x^2e^x dx = x^2e^x - \int 2xe^x dx$$

Using by parts again for $\int 2xe^x dx$

$$= 2xe^x - \int 2e^x dx = 2xe^x - 2e^x$$

$$\therefore \int x^2e^x dx = x^2e^x - [2xe^x - 2e^x]$$

$$= \boxed{x^2e^x - 2xe^x + 2e^x + C}$$

(e) **(6 pts)** $\int (1+sin(3x))^2 dx$. (Hint : You might need to use a trig identity)

$$\int \left(1 + \sin^2(3x) + 2\sin(3x)\right) dx$$

$$= \int 1\,dx + \int 2\sin(3x)\,dx + \int \sin^2(3x)\,dx$$

~~Using~~ $\sin^2 3x = \frac{1-\cos(6x)}{2}$

$$= x - \frac{2}{3}\cos(3x) + \int \frac{1-\cos(6x)}{2}\,dx$$

$$= x - \frac{2}{3}\cos(3x) + \frac{x}{2} - \frac{1}{12}\sin(6x) = \boxed{\frac{3x}{2} - \frac{2}{3}\cos(3x) - \frac{1}{12}\sin(6x) + C}$$

(f) **(6 pts)** $\int \frac{e^{-x}-1}{xe^{-x}-1} dx$. (Hint: Multiply the numerator and denominator by e^x)

$$\int \frac{e^{-x}-1}{xe^{-x}-1}\,dx = \int \frac{e^x(e^{-x}-1)}{e^x(xe^{-x}-1)}\,dx = \int \frac{1-e^x}{x-e^x}\,dx$$

Substitute $u = x - e^x$

$du = 1 - e^x$

$$\therefore \int \frac{1-e^x}{x-e^x}\,dx = \int \frac{1}{u}\,du = \ln|u| = \boxed{\ln|1-e^x| + C}$$

(g) **(6 pts)** $\int tan^2(x)dx$. (Hint : You might need to use a trig identity)

Use $\tan^2 x = \sec^2 x - 1$

$$\int \tan^2 x \, dx = \int (\sec^2 x - 1)\, dx$$

$$= \int \sec^2 x \, dx - \int 1 \, dx$$

$$= \boxed{\tan x - x + C}$$

(h) **(6 pts)** $\int \frac{x+2}{3-x}dx$.

$$\frac{x+2}{3-x} = -1 + \frac{5}{3-x}$$ using long division

$$\Rightarrow \int \frac{x+2}{3-x} dx = \int \left(-1 + \frac{5}{3-x}\right) dx$$

$$= \boxed{-x - 5\ln|3-x| + C}$$

2. Consider the region bounded by the graphs of $y = 2x^{1/3}$. $y = 0$ and $x = 1$.

(a) **(9 pts)** Compute the AREA of this region.

AREA $= \displaystyle\int_0^1 2x^{1/3}\,dx = \frac{2x^{4/3}}{4/3}\Big|_0^1$

$= \boxed{\frac{3}{2}}$

(b) **(6 pts)** Set up (but DO NOT EVALUATE) the integral to compute the volume of the solid obtained by revolving this region around the x-axis.

$\pi \displaystyle\int_0^1 (2x^{1/3})^2\,dx = \boxed{\pi \int_0^1 4x^{2/3}\,dx}$

3. **(10 pts)** Consider the region bounded by the curves $y = x^3 - 4x^2 + 1$ and $y = x - 3$. Set up (but DO NOT EVALUATE) the integral(s) to compute the area of this region.

Find intersection pts : $x^3 - 4x^2 + 1 = x - 3$

$\Rightarrow x^3 - 4x^2 - x + 4 = 0$

$\Rightarrow x^2(x-4) - (x-4) = 0$

$\Rightarrow (x^2-1)(x-4) = 0$

$\Rightarrow (x-1)(x+1)(x-4) = 0$

$\Rightarrow x = -1, 1, 4$

In $[-1, 1]$, $x^3 - 4x^2 + 1$ is above

In $[1, 4]$, $x - 3$ is above.

So area $= \boxed{\int_{-1}^{1} (x^3 - 4x^2 + 1) - (x-3)\, dx + \int_{1}^{4} (x-3) - (x^3 - 4x^2 + 1)\, dx}$

4. **(10 pts)** The temperature of a popsicle as a function of time t ($t = 0$ being when it is removed from the freezer) is given by

$$T(t) = \frac{30t}{t^2 + 1}$$

for $0 \leq t \leq 2$. What is the average temperature over this time period?.

Avg. $= \frac{1}{2-0} \int_0^2 \frac{30t}{t^2+1}\, dt$

Use substitution

$t^2 + 1 = u$

$2t\, dt = du$

$\int \frac{30t}{t^2+1}\, dt = \int \frac{15\, du}{u} = 15 \ln|u|$

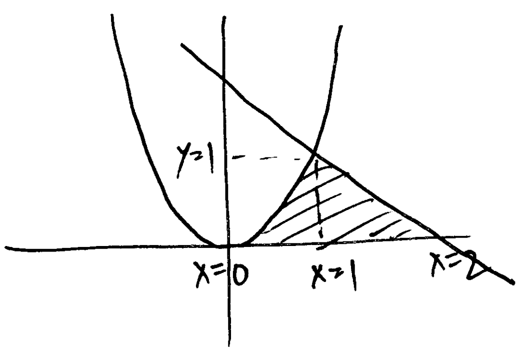
$= 15 \ln|t^2+1|$

$\Rightarrow$ Avg $= \frac{1}{2}\left[15 \ln|t^2+1| \Big|_0^2 \right] = \boxed{\frac{15}{2}(\ln 5 - \ln 1)}$

5. **(10 pts)** Consider the region bounded by the graphs $y = e^x$, $y = 1 - x$, $x = 0$ and $x = 1$. Set up (but DO NOT EVALUATE) the integral to compute the volume of the solid obtained by revolving this region around the x-axis.

Volume = $\pi \int_0^1 \left((e^x)^2 - (1-x)^2 \right) dx$

6. **(15 pts)** Consider the region bounded by the graphs of $y = x^2$, $y = 2 - x$, and $y = 0$. Set up (but DO NOT EVALUATE) the integral(s) to compute the volume of the solid obtained by revolving this region about the x axis.



Need to break up into 2 integrals.

$\pi \int_0^1 (x^2)^2 \, dx + \pi \int_1^2 (2-x)^2 \, dx$

$= \boxed{\pi \int_0^1 x^4 \, dx + \pi \int_1^2 (2-x)^2 \, dx}$