HW 6 Solutions MAT 17B

(2.) $\displaystyle\int 3x^2\sqrt{x^3+1}\,dx$

1. Substitute $u = x^3+1$

2. $\dfrac{du}{dx} = 3x^2 \Rightarrow 3x^2dx = du.$

3. $\displaystyle\int 3x^2\sqrt{x^3+1}\,dx = \int \underbrace{\sqrt{x^3+1}}_{u}\;\underbrace{3x^2dx}_{du} = \int \sqrt{u}\,du$

4. $\displaystyle\int \sqrt{u}\,du = \frac{2}{3}u^{3/2}$

5. $\displaystyle\int 3x^2\sqrt{x^3+1}\,dx = \boxed{\frac{2}{3}(x^3+1)^{3/2} + C}$

(6.) $\displaystyle\int 5\sin(1-2x)\,dx$

1. Substitute $u = 1-2x$

2. $\dfrac{du}{dx} = -2 \Rightarrow dx = \dfrac{-1}{2}du$

3. $\displaystyle\int 5\sin(1-2x)\,dx = \int 5\sin u\left(\frac{-1}{2}du\right) = \int \frac{-5}{2}\sin u\,du$

4. $\displaystyle\int \frac{-5}{2}\sin u\,du = -\frac{5}{2}(-\cos u) = \frac{5\cos u}{2}$

5. $\int 5\sin(1-2x)\,dx = \boxed{\frac{5}{2}\cos(1-2x) + C}$

(7) $\int 7x^2 \sin(4x^3)\,dx$

1. Substitute $u = 4x^3$
2. $\frac{du}{dx} = 12x^2 \Rightarrow \frac{1}{12}du = x^2dx$
3. $\int 7x^2 \sin(4x^3)\,dx = \int 7\sin(4x^3)\,x^2dx$

$$= \int 7\sin u\left(\frac{1}{12}du\right) = \int \frac{7}{12}\sin u\,du$$

4. $\int \frac{7}{12}\sin u\,du = -\frac{7}{12}\cos u$
5. $\int 7x^2 \sin(4x^3)\,dx = \boxed{-\frac{7}{12}\cos(4x^3) + C}$

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(9) $\int e^{2x+3}dx$

1. Substitute $u = 2x+3$
2. $\frac{du}{dx} = 2 \Rightarrow \frac{1}{2}du = dx$
3. $\int e^{2x+3}dx = \int e^u\left(\frac{1}{2}du\right) = \int \frac{1}{2}e^u du$
4. $\int \frac{1}{2}e^u du = \frac{1}{2}e^u$
5. $\int e^{2x+3}dx = \boxed{\frac{1}{2}e^{2x+3} + C}$

11. $\int x e^{-x^2/2}\,dx$

1. Substitute $u = -x^2/2$
2. $\frac{du}{dx} = -x \Rightarrow du = -x\,dx \Rightarrow -du = x\,dx$
3. $\int x e^{-x^2/2}\,dx = \int e^u(-du) = -\int e^u\,du$
4. $-\int e^u\,du = -e^u$
5. $\int x e^{-x^2/2}\,dx = \boxed{-e^{-x^2/2} + C}$

12. $\int x e^{1-3x^2}\,dx$

1. Substitute $u = 1-3x^2$
2. $\frac{du}{dx} = -6x \Rightarrow -\frac{1}{6}du = x\,dx$
3. $\int x e^{1-3x^2}\,dx = \int e^u\left(-\frac{1}{6}du\right) = -\frac{1}{6}\int e^u\,du$
4. $-\frac{1}{6}\int e^u\,du = -\frac{1}{6}e^u$
5. $\int x e^{1-3x^2}\,dx = \boxed{-\frac{1}{6}e^{1-3x^2} + C}$

(14) $\int \frac{2x}{3-x^2} dx$

1. $u = 3 - x^2$

2. $\frac{du}{dx} = -2x$

$\Rightarrow \quad -du = 2x\,dx$

3. $\int \frac{2x}{3-x^2} dx = \int \frac{1}{3-x^2} \cdot 2x\,dx = \int \frac{1}{u}(-du)$

4. $\int \frac{1}{u}(-du) = \int -\frac{1}{u} du = -\ln|u|$

5. $\int \frac{2x}{3-x^2} dx = \boxed{-\ln|3-x^2| + C}$

(15) $\int \frac{3x}{x+4} dx$

1. $u = x+4$

2. $\frac{du}{dx} = 1 \quad \Rightarrow \quad du = dx$

3. Express $x = u - 4$ $(\because u = x+4)$

So $\int \frac{3x}{x+4} dx = \int \frac{3(u-4)}{u} du$

4. $\int \frac{3(u-4)}{u} du = 3\int \frac{u-4}{u} du = 3\int \left(1 - \frac{4}{u}\right) du$

$= 3u - 4\ln|u|$

5. $\int \frac{3x}{x+4} dx = \boxed{3(x+4) - 4\ln|x+4| + C}$

(18) $\int (4-x)^{1/7}\,dx$

1. Substitute $u = 4 - x$

2. $\frac{du}{dx} = -1 \Rightarrow -du = dx$

3. $\int (4-x)^{1/7}\,dx = \int u^{1/7}(-du) = -\int u^{1/7}\,du$

4. $-\int u^{\frac{1}{7}}\,du = -\frac{u^{\frac{1}{7}+1}}{\frac{1}{7}+1} = -\frac{7u^{8/7}}{8}$

5. $\int (4-x)^{1/7}\,dx = \boxed{-\frac{7}{8}(4-x)^{8/7} + C}$

(21) $\int \frac{x-1}{1+4x-2x^2}\,dx$

1. Substitute $u = 1+4x-2x^2$

2. $\frac{du}{dx} = 4 - 4x = 4(1-x) \Rightarrow \frac{1}{4}du = (1-x)\,dx$

$-\frac{1}{4}du = (x-1)\,dx$

3. $\cancel{\int \frac{x-1}{1+4x-2x^2}\,dx = \int \frac{1}{1+4x-2x^2}(x-1)\,dx}$

3. $\int \frac{x-1}{1+4x-2x^2}\,dx = \int \frac{1}{1+4x-2x^2}(x-1)\,dx = \int \frac{1}{u}\left(-\frac{1}{4}du\right)$

$= -\frac{1}{4}\int \frac{1}{u}\,du$

4. $\frac{-1}{4}\int \frac{1}{u}\,du = \frac{-1}{4}\ln|u|$

5. $\int \frac{x-1}{1+4x-2x^2}\,dx = \boxed{-\frac{1}{4}\ln|1+4x-2x^2| + C}$

25. $\int 3xe^{x^2}\,dx$

1. $u = x^2$
2. $\frac{du}{dx} = 2x \Rightarrow \frac{1}{2}du = x\,dx$
3. $\int 3xe^{x^2}dx = \int 3e^{x^2}x\,dx$

$= \int 3e^{u}\left(\frac{1}{2}du\right)$

$= \frac{3}{2}\int e^{u}du$

4. $\frac{3}{2}\int e^{u}du = \frac{3}{2}e^{u}$
5. $\int 3xe^{x^2}dx = \boxed{\frac{3}{2}e^{x^2} + C}$

27. $\int \frac{1}{x} \csc^2(\ln x)\, dx$

1. $u = \ln x$
2. $\frac{du}{dx} = \frac{1}{x} \Rightarrow du = \frac{1}{x} dx$
3. $\int \frac{1}{x} \csc^2(\ln x)\, dx = \int \csc^2(u)\, du$
4. $\int \csc^2(u)\, du = -\cot(u)$
5. $\int \frac{1}{x} \csc^2(\ln x)\, dx = \boxed{-\cos(\ln x) + C}$

28. $\int \sec^2 x\, e^{\tan x}\, dx$

1. $u = \tan x$
2. $\frac{du}{dx} = \sec^2 x \Rightarrow du = \sec^2 x\, dx$
3. $\int \sec^2 x\, e^{\tan x}\, dx = \int e^u\, du$
4. $\int e^u\, du = e^u$
5. $\int \sec^2 x\, e^{\tan x}\, dx = \boxed{e^{\tan x} + C}$

32. $\int \sin^3 x \cos x\, dx$

1. $u = \sin x$
2. $\frac{du}{dx} = \cos x \Rightarrow du = \cos x\, dx$
3. $\int \sin^3 x \cos x\, dx = \int u^3\, du$
4. $\int u^3\, du = \frac{u^4}{4}$
5. $\int \sin^3 x \cos x\, dx = \boxed{(\sin^4 x) + C}$

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29. $\int \sin\left(\frac{3\pi}{2}x + \frac{\pi}{4}\right) dx$

1. Substitute $u = \frac{3\pi}{2}x + \pi/4$
2. $\frac{du}{dx} = \frac{3\pi}{2} \Rightarrow \frac{2}{3\pi}du = dx$
3. $\int \sin\left(\frac{3\pi}{2}x + \frac{\pi}{4}\right) dx = \int (\sin u)\left(\frac{2}{3\pi}du\right)$
4. $\int (\sin u)\left(\frac{2}{3\pi}du\right) = \frac{2}{3\pi}\int \sin u \, du = -\frac{2}{3\pi}\cos u$
5. $\int \sin\left(\frac{3\pi}{2}x + \frac{\pi}{4}\right) dx = \boxed{-\frac{2}{3\pi}\cos\left(\frac{3\pi}{2}x + \frac{\pi}{4}\right) + C}$

30. $\int \cos(2x-1)\, dx$

1. Substitute $u = 2x-1$
2. $\frac{du}{dx} = 2 \Rightarrow \frac{1}{2}du = dx$
3. $\int \cos(2x-1)dx = \int \cos(u)\left(\frac{1}{2}du\right)$
4. $\int \cos u\left(\frac{1}{2}du\right) = \frac{1}{2}\int \cos u \, du = \frac{1}{2}\sin u$
5. $\int \cos(2x-1)dx = \boxed{\frac{1}{2}\sin(2x-1) + C}$

1. $u = 5+x^2$

2. $du = 2x\,dx \Rightarrow \frac{1}{2}du = x\,dx$

3. $\int x^3\sqrt{5+x^2} = \int x^2\sqrt{5+x^2}\; x\,dx$

$(u = 5+x^2 \Rightarrow x^2 = u-5)$

so $\int x^2\sqrt{5+x^2}\; x\,dx = \int (u-5)\sqrt{u}\left(\frac{1}{2}du\right)$

4. $\int (u-5)\sqrt{u}\left(\frac{1}{2}du\right) = \frac{1}{2}\int (u-5)\sqrt{u}\,du = \frac{1}{2}\left(\int u\sqrt{u} - 5\sqrt{u}\,du\right)$

$= \frac{1}{2}\left(\frac{2u^{5/2}}{5}\right) - \frac{5}{2}\left(\frac{2}{3}u^{3/2}\right)$

$= \frac{u^{5/2}}{5} - \frac{5}{3}u^{3/2}$

5. $\int x^3\sqrt{5+x^2}\,dx = \boxed{\frac{(5+x^2)^{5/2}}{5} - \frac{5}{3}(5+x^2)^{3/2} + C}$

48. $\displaystyle\int_{\ln 4}^{\ln 7} \frac{e^x}{(e^x-3)^2}\,dx$

First compute $\displaystyle\int \frac{e^x}{(e^x-3)^2}\,dx$ + then put in limits.

$$\int \frac{e^x}{(e^x-3)^2}\,dx$$

1. $u = e^x - 3$
2. $\frac{du}{dx} = e^x \Rightarrow du = e^x\,dx$
3. $\displaystyle\int \frac{e^x}{(e^x-3)^2}\,dx = \int \frac{1}{(e^x-3)^2}\,e^x\,dx = \int \frac{1}{u^2}\,du$
4. $\displaystyle\int \frac{1}{u^2}\,du = -u^{-1} = -\frac{1}{u}$
5. $\displaystyle\int \frac{e^x}{(e^x-3)^2}\,dx = -1/(e^x-3)$

Put in limits:

$$\int_{\ln 4}^{\ln 7} \frac{e^x}{(e^x-3)^2}\,dx = \left.\frac{-1}{e^x-3}\right|_{\ln 4}^{\ln 7} = \frac{-1}{e^{\ln 7}-3} - \left(\frac{-1}{e^{\ln 4}-3}\right)$$

$$= \frac{-1}{7-3} - \left(\frac{-1}{4-3}\right)$$

$$= -\frac{1}{4} + 1 = \frac{3}{4}$$

56. $$\int_1^2 \frac{x\,dx}{(x^2+1)\ln(x^2+1)}$$

First compute $$\int \frac{x\,dx}{(x^2+1)\ln(x^2+1)}.$$

There are 2 ways of doing this indefinite integral.

WAY 1: 1. $u = x^2+1$

2. $\frac{du}{dx} = 2x \Rightarrow \frac{1}{2}du = x\,dx$

3. $$\int \frac{x\,dx}{(x^2+1)\ln(x^2+1)} = \int \frac{1}{u} \cdot \frac{1}{\ln(u)} \left(\frac{1}{2}du\right)$$

4. $$\int \frac{1}{u\ln u}\left(\frac{1}{2}du\right) = \frac{1}{2}\int \frac{du}{u\ln u}$$

Use substitution again!

1. $V = \ln u$

2. $\frac{dv}{du} = \frac{1}{u} \Rightarrow dv = \frac{1}{u}du$

3. $$\frac{1}{2}\int \frac{du}{u\ln u} = \frac{1}{2}\int \frac{1}{\ln u} \cdot \frac{1}{u}du$$

$$= \frac{1}{2}\int \frac{1}{v}dv = \frac{1}{2}\ln|v|$$

5. $$\frac{1}{2}\int \frac{du}{u\ln u} = \frac{1}{2}\ln|\ln(u)|$$

5. $\int \frac{x\,dx}{(x^2+1)\ln(x^2+1)} = \boxed{\frac{1}{2}\ln\left|\ln(x^2+1)\right| + C}$.

WAY 2 :

1. Make a different substitution :

$$u = \ln(x^2+1)$$

2. $\frac{du}{dx} = \frac{1}{x^2+1} \cdot 2x \Rightarrow \frac{1}{2}du = \frac{x}{x^2+1}dx$

3. $\int \frac{x\,dx}{(x^2+1)\ln(x^2+1)} = \int \frac{1}{\ln(x^2+1)}\left(\frac{x}{x^2+1}dx\right)$

$$= \int \frac{1}{u}\left(\frac{1}{2}du\right)$$

4. $\int \frac{1}{u}\left(\frac{1}{2}du\right) = \frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\ln|u|$

5. $\int \frac{x\,dx}{(x^2+1)\ln(x^2+1)} = \boxed{\frac{1}{2}\ln\left|\ln(x^2+1)\right| + C}$

Finally, put in limits :

$$\int_1^2 \frac{x\,dx}{(x^2+1)\ln(x^2+1)} = \frac{1}{2}\ln\left|\ln(2^2+1)\right| - \frac{1}{2}\ln\left|\ln(1^2+1)\right|$$

$$= \boxed{\frac{1}{2}\ln(\ln 5) - \frac{1}{2}\ln(\ln 2)}$$