

41.

$$\text{Rotation matrix : } \begin{bmatrix} \cos \pi/6 & -\sin \pi/6 \\ \sin \pi/6 & \cos \pi/6 \end{bmatrix}$$

$$\text{New vector : } \begin{bmatrix} \cos \pi/6 & -\sin \pi/6 \\ \sin \pi/6 & \cos \pi/6 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -\cos \pi/6 - 2\sin \pi/6 \\ -\sin \pi/6 + 2\cos \pi/6 \end{bmatrix}$$

42.

$$\text{Rotation matrix : } \begin{bmatrix} \cos \pi/3 & -\sin \pi/3 \\ \sin \pi/3 & \cos \pi/3 \end{bmatrix}$$

$$\text{New vector : } \begin{bmatrix} \cos \pi/3 & -\sin \pi/3 \\ \sin \pi/3 & \cos \pi/3 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4\cos \pi/3 + \sin \pi/3 \\ 4\sin \pi/3 - \cos \pi/3 \end{bmatrix}$$

45. Rotating CLOCKWISE means angle θ in
rotation matrix is $-\pi/4$.

$$\therefore \begin{bmatrix} \cos(-\pi/4) & -\sin(-\pi/4) \\ \sin(-\pi/4) & \cos(-\pi/4) \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$\text{New vector} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

46. CLOCKWISE \equiv angle $\theta = -\pi/3$

$$\text{Rotation matrix} : \begin{bmatrix} \cos(-\pi/3) & -\sin(-\pi/3) \\ \sin(-\pi/3) & \cos(-\pi/3) \end{bmatrix}$$

$$\text{New vector} : \begin{bmatrix} \cos(-\pi/3) & -\sin(-\pi/3) \\ \sin(-\pi/3) & \cos(-\pi/3) \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(-\pi/3) + 2\sin(-\pi/3) \\ \sin(-\pi/3) - 2\cos(-\pi/3) \end{bmatrix}$$