

9.

$$2x - y = 3 \quad (1)$$

$$(x - 3y = 7) \times (-2)$$

$$\Downarrow$$

$$2x - y = 3$$

$$+5y = -11$$

$$\Rightarrow \underline{y = -11/5}, \quad x = \frac{3+y}{2} = \frac{3 - 11/5}{2} = \underline{\underline{2/5}}$$

10.

$$(5x - 3y = 2) \times (2)$$

$$(2x + 7y = 3) \times (-5)$$

$$\Downarrow$$

$$5x - 3y = 2$$

$$-41y = -11$$

$$\Rightarrow \underline{y = 11/41} \quad x = \frac{2+3y}{5} = \frac{2+3(11/41)}{5} = \underline{\underline{23/41}}$$

13.

$$(3x - y = 1) \times 1$$

$$(-3x + y = 4) \times 1$$

$$\Downarrow$$

$$3x - y = 1$$

$$0 = 5$$

$$\Rightarrow \underline{\underline{\text{no solutions}}}, \quad \underline{\underline{\text{inconsistent}}}$$

16.

$$(x - 2y = 2) \times 2$$

$$(-2x + 4y = -4) \times 1$$

$$\begin{array}{r} \downarrow \\ x - 2y = 2 \\ 0 = 0 \end{array}$$

$\Rightarrow$  infinitely many solutions

17.

$x = \#$  of fish,  $y = \#$  of plants.

$$\therefore (x + y = 11) \times (-2 \cdot 30)$$

$$(2 \cdot 30x + 1 \cdot 70y = 21 \cdot 70) \times 1$$

$\downarrow$

$$x + y = 11$$

$$-0.6y = \cancel{21.70} - (11 \times 2 \cdot 30) = 21.7 - 25.3 = -3.6$$

$$\therefore y = \cancel{6} = 6 \quad x = 11 - y = \cancel{5} = 5$$

20.

$$(2x - 3y + z = -1) \times 1$$

$$(x + y - 2z = -3) \times (-2)$$

$$3x - 2y + z = 2$$

$\downarrow$

$$(2x - 3y + z = -1) \times 3$$

$$-5y + 5z = 5$$

$$(3x - 2y + z = 2) \times (-2)$$

$\Downarrow$ 

$$2x - 3y + z = -1$$

$$(-5y + 5z = 5) \times (-1)$$

$$(-5y - z = -7) \times 1$$

 $\Downarrow$ 

$$2x - 3y + z = -1$$

$$-5y + 5z = 5$$

$$-6z = -12$$

$$\Rightarrow \boxed{z = 2, y = \frac{5 - 5z}{-5} = 1, x = \frac{-1 - z + 3y}{2} = 0}$$

21.

$$(5x - y + 2z = 6) \times 1$$

$$(x + 2y - z = -1) \times (-5)$$

$$3x + 2y - 2z = 1$$

 $\Downarrow$ 

$$(5x - y + 2z = 6) \times 3$$

$$-11y + 7z = 11$$

$$(3x + 2y - 2z = 1) \times (-5)$$

 $\Downarrow$ 

$$5x - y + 2z = 6$$

$$(-11y + 7z = 11) \times 13$$

$$(-13y + 16z = 13) \times (-11)$$

$$\Downarrow$$

$$5x - y + 2z = 6$$

$$-11y + 7z = 11$$

$$-85z = 0$$

$$\Rightarrow \boxed{z = 0, \quad y = \frac{11 - 7z}{-11} = -1, \quad x = \frac{6 - 2z + y}{5} = 1}$$

23.

$$(-2x + 4y - z = -1) \times 1$$

$$(x + 7y + 2z = -4) \times 2$$

$$3x - 2y + 3z = -3$$

$$\Downarrow$$

$$(-2x + 4y - z = -1) \times 3$$

$$+18y + 3z = -9$$

$$(3x - 2y + 3z = -3) \times 2$$

$$\Downarrow$$

$$-2x + 4y - z = -1$$

$$(+18y + 3z = -9) \times 8$$

$$(+8y + 3z = -9) \times (-18)$$

$$\Downarrow$$

$$-2x + 4y - z = -1$$

$$+18y + 3z = -9$$

$$-30z = 90$$

$\Rightarrow$

$$\boxed{z = -3}$$

$$y = \frac{-9 - 3z}{18} = 0$$

$$x = \frac{-1 + z - 4y}{-2} = +2$$

25.

Augmented matrix :

$$\begin{array}{l} 2 \times \\ 1 \times \end{array} \left[ \begin{array}{ccc|c} -1 & -2 & 3 & -9 \\ 2 & 1 & -1 & 5 \\ 4 & -3 & 5 & -9 \end{array} \right]$$

$\Downarrow$

$$\begin{array}{l} 4 \times \\ 1 \times \end{array} \left[ \begin{array}{ccc|c} -1 & -2 & 3 & -9 \\ 0 & -3 & 5 & -13 \\ 4 & -3 & 5 & -9 \end{array} \right]$$

$\Downarrow$

$$\begin{array}{l} 11 \times \\ (-3) \times \end{array} \left[ \begin{array}{ccc|c} -1 & -2 & 3 & -9 \\ 0 & -3 & 5 & -13 \\ 0 & -11 & 17 & -45 \end{array} \right]$$

$\Downarrow$

$$\left[ \begin{array}{ccc|c} -1 & -2 & 3 & -9 \\ 0 & -3 & 5 & -13 \\ 0 & 0 & 4 & -8 \end{array} \right]$$

$$\Rightarrow \begin{array}{l} -x - 2y + 3z = -9 \\ -3y + 5z = -13 \\ 4z = -8 \end{array}$$

$\Rightarrow$

$$\begin{array}{l} z = -2 \\ y = \frac{-13 - 5z}{-3} \\ = 1 \\ x = \frac{-9 - 3z + 2y}{-1} \\ = 1 \end{array}$$

26.

Augmented matrix

$$4 \times \left[ \begin{array}{ccc|c} 3 & -2 & 1 & 4 \\ (-3) \times & 4 & 1 & -2 \\ & 2 & -3 & 1 & 7 \end{array} \right]$$

$$\Downarrow$$

$$2 \times \left[ \begin{array}{ccc|c} 3 & -2 & 1 & 4 \\ & 0 & -11 & 10 & 52 \\ (-3) \times & 2 & -3 & 1 & 7 \end{array} \right]$$

$$\Downarrow$$

$$5 \times \left[ \begin{array}{ccc|c} 3 & -2 & 1 & 4 \\ & 0 & -11 & 10 & 52 \\ 11 \times & 0 & 5 & -1 & -13 \end{array} \right]$$

$$\Downarrow$$

$$\left[ \begin{array}{ccc|c} 3 & -2 & 1 & 4 \\ & 0 & -11 & 10 & 52 \\ & 0 & 0 & 39 & 117 \end{array} \right]$$

so

$$\begin{aligned} 3x - 2y + z &= 4 \\ -11y + 10z &= 52 \\ 39z &= 117 \end{aligned}$$

$$\Rightarrow$$

$$z = \frac{117}{39} = 3$$

$$y = \frac{52 - 10z}{-11} = -2$$

$$x = \frac{4 - z + 2y}{3} = -1$$

28.

Augmented Matrix

$$1 \times \left[ \begin{array}{ccc|c} 2 & 0 & -1 & 1 \\ 2 \times \left[ \begin{array}{ccc|c} -1 & 1 & 3 & -1 \\ 1 & -1 & 1 & -3 \end{array} \right] \end{array} \right]$$

$$\Downarrow$$

$$1 \times \left[ \begin{array}{ccc|c} 2 & 0 & -1 & 1 \\ 0 & 2 & 5 & -1 \\ (-2) \times \left[ \begin{array}{ccc|c} 1 & -1 & 1 & -3 \end{array} \right] \end{array} \right]$$

$$\Downarrow$$

$$1 \times \left[ \begin{array}{ccc|c} 2 & 0 & -1 & 1 \\ 0 & 2 & 5 & -1 \\ (-1) \times \left[ \begin{array}{ccc|c} 0 & 2 & -3 & 7 \end{array} \right] \end{array} \right]$$

$$\Downarrow$$

$$\left[ \begin{array}{ccc|c} 2 & 0 & -1 & 1 \\ 0 & 2 & 5 & -1 \\ 0 & 0 & 8 & -8 \end{array} \right]$$

$$\Rightarrow \begin{aligned} 2x - z &= 1 \\ 2y + 5z &= -1 \\ 8z &= -8 \end{aligned} \Rightarrow$$

$$\begin{aligned} z &= -1 \\ y &= \frac{-1 - 5z}{2} = -3 \\ x &= \frac{1 + z}{2} = 0 \end{aligned}$$