...ology At any time t, the rate of growth of the popula- ...N of deer in a state park is proportional to the product ...N and $L - N$, where $L = 500$ is the maximum number ...deer the park can maintain. When $t = 0$, $N = 100$, and ...$t = 4$, $N = 200$. Write N as a function of t.

...les Growth The rate of change in sales S (in thou-...s of units) of a new product is proportional to the ...ct of S and $L - S$. L (in thousands of units) is the esti-... maximum level of sales, and $S = 10$ when $t = 0$. ...e and solve the differential equation for this sales ...del.

...g Theory In Exercises 17 and 18, assume that the ...change in the proportion P of correct responses after n ...proportional to the product of P and $L - P$, where L is ...g proportion of correct responses.

...e and solve the differential equation for this learning ...y model.

...e the solution of Exercise 17 to write P as a function of ... then use a graphing utility to graph the solution.

$L = 1.00$

$P = 0.50$ when $n = 0$

$P = 0.85$ when $n = 4$

$L = 0.80$

$P = 0.25$ when $n = 0$

$P = 0.60$ when $n = 10$

...al Reaction In Exercises 19 and 20, use the chemical ... model in Example 2 to find the amount y as a function ...use a graphing utility to graph the function.

...45 grams when $t = 0$; $y = 4$ grams when $t = 2$

...75 grams when $t = 0$; $y = 12$ grams when $t = 1$

...cises 21 and 22, use the Gompertz growth model ...d in Example 3 to find the growth function, and sketch ...h.

...500; $y = 100$ when $t = 0$; $y = 150$ when $t = 2$

...5000; $y = 500$ when $t = 0$; $y = 625$ when $t = 1$

...logy A population of eight beavers has been intro-...ced into a new wetlands area. Biologists estimate that the ...imum population the wetlands can sustain is 60 ...vers. After 3 years, the population is 15 beavers. If the ...pulation follows a Gompertz growth model, how many ...vers will be in the wetlands after 10 years?

...logy A population of 30 rabbits has been introduced ... a new region. It is estimated that the maximum popu-...on the region can sustain is 400 rabbits. After 1 year, the ...pulation is estimated to be 90 rabbits. If the population ...ows a Gompertz growth model, how many rabbits will ... present after 3 years?

Biology In Exercises 25 and 26, use the hybrid selection model in Example 4 to find the percent of the population that has the indicated characteristic.

25. You are studying a population of mayflies to determine how quickly characteristic A will pass from one generation to the next. At the start of the study, half the population has characteristic A. After four generations, 75% of the population has characteristic A. Find the percent of the population that will have characteristic A after 10 generations. (Assume $a = 2$ and $b = 1$.)

26. A research team is studying a population of snails to determine how quickly characteristic B will pass from one generation to the next. At the start of the study, 40% of the snails have characteristic B. After five generations, 80% of the population has characteristic B. Find the percent of the population that will have characteristic B after eight generations. (Assume $a = 2$ and $b = 1$.)

27. ***Chemical Reaction*** In a chemical reaction, a compound changes into another compound at a rate proportional to the unchanged amount, according to the model

$$\frac{dy}{dt} = ky.$$

(a) Solve the differential equation.

(b) If the initial amount of the original compound is 20 grams, and the amount remaining after 1 hour is 16 grams, when will 75% of the compound have been changed?

28. ***Chemical Mixture*** A 100-gallon tank is full of a solution containing 25 pounds of a concentrate. Starting at time $t = 0$, distilled water is admitted to the tank at the rate of 5 gallons per minute, and the well-stirred solution is withdrawn at the same rate.

(a) Find the amount Q of the concentrate in the solution as a function of t. (*Hint:* $Q' + Q/20 = 0$)

(b) Find the time when the amount of concentrate in the tank reaches 15 pounds.

29. ***Chemical Mixture*** A 200-gallon tank is half full of distilled water. At time $t = 0$, a solution containing 0.5 pound of concentrate per gallon enters the tank at the rate of 5 gallons per minute, and the well-stirred mixture is withdrawn at the same rate. Find the amount Q of concentrate in the tank after 30 minutes. (*Hint:* $Q' + Q/20 = \frac{5}{2}$)

30. ***Safety*** Assume that the rate of change in the number of miles s of road cleared per hour by a snowplow is inversely proportional to the depth h of snow. That is,

$$\frac{ds}{dh} = \frac{k}{h}.$$

Find s as a function of h if $s = 25$ miles when $h = 2$ inches and $s = 12$ miles when $h = 6$ inches ($2 \le h \le 15$).

31. ***Chemistry*** A wet towel hung from a clothesline to dry loses moisture through evaporation at a rate proportional to its moisture content. If after 1 hour the towel has lost 40% of its original moisture content, after how long will it have lost 80%?

32. ***Biology*** Let x and y be the sizes of two internal organs of a particular mammal at time t. Empirical data indicate that the relative growth rates of these two organs are equal, and can be modeled by

$$\frac{1}{x}\frac{dx}{dt} = \frac{1}{y}\frac{dy}{dt}.$$

Use this differential equation to write y as a function of x.

33. ***Population Growth*** When predicting population growth, demographers must consider birth and death rates as well as the net change caused by the difference between the rates of immigration and emigration. Let P be the population at time t and let N be the net increase per unit time due to the difference between immigration and emigration. So, the rate of growth of the population is given by

$$\frac{dP}{dt} = kP + N, \quad N \text{ is constant.}$$

Solve this differential equation to find P as a function of time.

34. ***Meteorology*** The barometric pressure y (in inches of mercury) at an altitude of x miles above sea level decreases at a rate proportional to the current pressure according to the model

$$\frac{dy}{dx} = -0.2y$$

where $y = 29.92$ inches when $x = 0$. Find the barometric pressure (a) at the top of Mt. St. Helens (8364 feet) and (b) at the top of Mt. McKinley (20,320 feet).

35. ***Investment*** A large corporation starts at time $t = 0$ to invest part of its receipts at a rate of P dollars per year in a fund for future corporate expansion. Assume that the fund earns r percent interest per year compounded continuously. So, the rate of growth of the amount A in the fund is given by

$$\frac{dA}{dt} = rA + P$$

where $A = 0$ when $t = 0$. Solve this differential equation for A as a function of t.

Investment In Exercises 36–38, use the result of Exercise 35.

36. Find A for each situation.

(a) $P = \$100{,}000$, $r = 12\%$, and $t = 5$ years

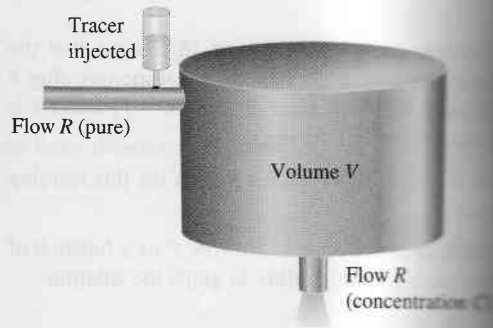
(b) $P = \$250{,}000$, $r = 15\%$, and $t = 10$ years

37. Find P if the corporation needs $120,000,000 in 8 years and the fund earns $16\frac{1}{4}\%$ interest compounded continuously.

38. Find t if the corporation needs $800,000 and … $75,000 per year in a fund earning 13% … pounded continuously.

Medical Science In Exercises 39–41, a medical … wants to determine the concentration C (in moles … tracer drug injected into a moving fluid. Solve … considering a single-compartment dilution model … Assume that the fluid is continuously mixed and … of fluid in the compartment is constant.

Figure for 39–41



39. If the tracer is injected instantaneously … the concentration of the fluid in the compartment … diluting according to the differential equation

$$\frac{dC}{dt} = \left(-\frac{R}{V}\right)C, \quad C = C_0 \text{ when } t = 0.$$

(a) Solve this differential equation to find the … tion as a function of time.

(b) Find the limit of C as $t \to \infty$.

40. Use the solution of the differential equation … to find the concentration as a function of time … graphing utility to graph the function.

(a) $V = 2$ liters, $R = 0.5$ L/min, and C_0 …

(b) $V = 2$ liters, $R = 1.5$ L/min, and C_0 …

41. In Exercises 39 and 40, it was assumed … gle initial injection of the tracer drug into the … Now consider the case in which the tracer is … injected (beginning at $t = 0$) at the rate of … Considering Q to be negligible compared … differential equation

$$\frac{dC}{dt} = \frac{Q}{V} - \left(\frac{R}{V}\right)C, \quad C = 0 \text{ when } t = 0$$

(a) Solve this differential equation to find … tion as a function of time.

(b) Find the limit of C as $t \to \infty$.