31. ~~Let~~ Let the moisture content at time t be $Q(t)$.

Rate of evaporation (i.e. $\frac{dQ}{dt}$) is proportional to current moisture content.

$$\therefore \quad \frac{dQ}{dt} \propto Q$$

Using a proportionality constant k.

$$\frac{dQ}{dt} = kQ$$

Solving this differential equation using separation of variables :

$$\int \frac{1}{Q}\,dQ = \int k\,dt$$

$$\ln|Q| = kt + C$$

Exponentiating both sides $\Rightarrow$ $|Q| = e^{(kt+C)} = e^{C}e^{kt}$

$$\Rightarrow \boxed{Q = \pm e^{C}e^{kt} = C_1 e^{kt}}$$

(Renaming $C_1 \equiv \pm e^{C}$)

After 1 hour (i.e. $t=1$), 40% moisture is gone.

So 60% of moisture is left.

First, lets see how much moisture is there at the begining ($t=0$).

$$Q(0) = C_1 e^{k(0)} = C_1$$

$\therefore$ we have C_1 amount of moisture to begin with & 60% is left after $t=1$.

$$\therefore \frac{60}{100} C_1 = Q(1).$$

$$\Rightarrow \frac{60}{100} \cancel{C_1} = \cancel{C_1} e^{k(1)}$$ [Plugging $t=1$ into our formula $Q(t) = C_1 e^{kt}$]

$$\Rightarrow \frac{60}{100} = e^k$$

$$\Rightarrow k = \ln\left(\frac{60}{100}\right) = \ln\left(\frac{6}{10}\right)$$

How long will it take to lose 80%?

$\therefore$ find t such that remaining moisture is 20% of the initial moisture.

$$\therefore \quad Q(t) = \frac{20}{100} C_1$$

$$\therefore \quad \frac{20}{100} \cancel{C_1} = C_1 e^{kt} = \cancel{C_1} e^{\ln\left(\frac{6}{10}\right) t}$$

$$\Rightarrow \quad \frac{1}{5} = e^{\ln\left(\frac{6}{10}\right) t}$$

$$\therefore \quad \ln\left(\frac{1}{5}\right) = \ln\left(e^{\ln\left(\frac{6}{10}\right) t}\right)$$

$$\Rightarrow \quad \ln\left(\frac{1}{5}\right) = \left(\ln\left(\frac{6}{10}\right)\right) t$$

$$\Rightarrow \quad \boxed{t = \frac{\ln\left(\frac{1}{5}\right)}{\ln\left(\frac{6}{10}\right)}}$$