HW 15 Solutions MAT 17B

2. $y' - 5(2x - y) = 0$

$\Rightarrow y' - 10x + 5y = 0$

$\Rightarrow \boxed{y' + 5y = 10x}$ Standard form.

$P(x) = 5, \quad Q(x) = 10x.$

4. $xy' + y = x^3 y$

$\Rightarrow xy' + y - x^3 y = 0$

$\Rightarrow xy' + (1 - x^3)y = 0$ (Divide through by x)

$\Rightarrow \boxed{y' + \frac{(1-x^3)}{x} y = 0}$ Standard form.

$P(x) = \frac{1-x^3}{x}, \quad Q(x) = 0.$

6. $x = x^2(y' + y)$

$\Rightarrow \frac{x}{x^2} = y' + y \Rightarrow \boxed{y' + y = \frac{1}{x}}$ Standard form.

$P(x) = 1 \quad Q(x) = \frac{1}{x}.$

10. $\frac{dy}{dx} + 3y = e^{-3x}$

$P(x) = 3 \qquad Q(x) = e^{-3x}$

1. $\int P(x)\,dx = \int 3\,dx = 3x$

2. $u(x) = e^{\int P(x)dx} = e^{3x}$

3. $\int Q(x)\,u(x)\,dx = \int (e^{-3x})(e^{3x})\,dx = \int 1\,dx$

$= x$

$\therefore$ Final Solution $y = \frac{1}{e^{3x}}(x + c)$

$y = \boxed{xe^{-3x} + Ce^{-3x}}$

12. $\frac{dy}{dx} + \frac{2y}{x} = 3x+1$

$P(x) = \frac{2}{x}, \quad Q(x) = 3x+1$

1. $\int P(x)\,dx = \int \frac{2}{x}\,dx = 2\ln|x|.$

2. $u(x) = e^{2\ln|x|} = |x|^2 = x^2$

3. $\int Q(x)\,u(x) = \int (3x+1)\,x^2\,dx$

$$= \int (3x^3 + x^2)\, dx$$

$$= \frac{3x^4}{4} + \frac{x^3}{3}$$

$\therefore$ Final Solution $y = \frac{1}{x^2}\left(\frac{3x^4}{4} + \frac{x^3}{3} + C\right)$

$$y = \boxed{\frac{3x^2}{4} + \frac{x}{3} + Cx^{-2}}$$

18.

$$xy' + y = x^2 \ln x$$

$$\Rightarrow \quad y' + \frac{y}{x} = \frac{x^2 \ln x}{x} = x \ln x$$

$$P(x) = \frac{1}{x}, \quad Q(x) = x \ln x$$

1. $\int P(x)\, dx = \int \frac{1}{x}\, dx = \ln|x|$

2. $u(x) = e^{\int P(x)\,dx} = e^{\ln|x|} = |x| = \pm x$

3. $\int Q(x)\, u(x)\, dx = \int x \ln x\, (\pm x)\, dx$

$$= \pm \int x^2 \ln x\, dx$$

Let's evaluate $\int x^2 \ln x\, dx$

Use integration by parts.

Using "LATE" : $f(x) = \ln x$, $g(x) = x^2$.

1. $\int g(x)\,dx = \int x^2\,dx = \frac{x^3}{3} = G(x)$

2. $\int f'(x)\,G(x)\,dx = \int \frac{1}{x}\cdot\frac{x^3}{3}\,dx = \int \frac{x^2}{3}\,dx$

$$= \frac{x^3}{9}$$

3. $\int f(x)\,g(x)\,dx = (\ln x)\left(\frac{x^3}{3}\right) - \frac{x^3}{9}$

$\therefore$ Final Solution : $y = \frac{1}{u(x)}\left[\int Q(x)u(x)\,dx\right]$

$$= \frac{1}{\pm x}\left[\pm\left(\ln x\left(\frac{x^3}{3}\right) - \frac{x^3}{9}\right) + C\right]$$

$$= (\ln x)\left(\frac{x^2}{3}\right) - \frac{x^2}{9} \pm \frac{C}{x}$$

$$= (\ln x)\left(\frac{x^2}{3}\right) - \frac{x^2}{9} + \frac{C_1}{x}$$

(Renaming the $\pm C$ as C_1 just to write it nicely)

NOTE: Whenever you have

$$e^{\ln|x|} = |x| = \pm x$$

You can assume it as $+x$ as proceed the final answer will still be right You do not have to carry around the $\pm$

27. $$y' + y = 6e^x$$

$P(x) = 1$, $Q(x) = 6e^x$

1. $\int P(x)\,dx = \int 1\,dx = x$

2. $u(x) = e^{\int P(x)\,dx} = e^x$

3. $\int Q(x)\,u(x) = \int (6e^x)(e^x)\,dx = \int 6e^{2x}\,dx$

$$= \frac{6}{2}e^{2x} = 3e^{2x}$$

Final Solution:

$$y = \frac{1}{e^x}\left[3e^{2x} + C\right] = 3e^x + Ce^{-x}$$

Now, $y = 3$ when $x = 0$

$\therefore \quad 3 = 3e^0 + Ce^{-0} = 3 + C$

$\Rightarrow \quad C = 0.$

$\therefore \quad \boxed{y = 3e^x}$

31. $y' + 3x^2 y = 3x^2$

$P(x) = 3x^2, \quad Q(x) = 3x^2$

1. $\int P(x)\,dx = \int 3x^2\,dx = x^3$

2. $u(x) = e^{\int P(x)\,dx} = e^{x^3}$

3. $\int Q(x)\,u(x)\,dx = \int (3x^2)(e^{x^3})\,dx$

Use u - substitution

$u = x^3$

$\frac{du}{dx} = 3x^2 \Rightarrow du = 3x^2 dx$

$\Rightarrow \int (3x^2) e^{x^3} dx = \int e^u\,du$

$= e^u$

$= e^{x^3}$

$\therefore$ Final Solution :

$$y = \frac{1}{e^{x^3}}\left[e^{x^3} + C\right]$$

$$= 1 + Ce^{-x^3}$$

Using $\underline{y = 6 \quad \text{when} \quad x = 0}$

$$6 = 1 + Ce^{-x^3} = 1 + Ce^{-(0)^3}$$

$$= 1 + C$$

$$\Rightarrow \quad C = 5.$$

$$\therefore \boxed{y = 1 + 5e^{-x^3}}$$

$\underline{32.}$ $\quad y' + (2x-1)y = 0$

$P(x) = 2x-1 \quad , \quad Q(x) = 0.$

1. $\int P(x)\,dx = \int (2x-1)\,dx = x^2 - x$

2. $u(x) = e^{\int P(x)\,dx} = e^{(x^2-x)}$

3. $\int Q(x)\,u(x)\,dx = \int 0 \cdot e^{(x^2-x)}\,dx = \int 0\,dx$

$$= \underline{\underline{0}}.$$

Final Solution :

$$y = \frac{1}{e^{(x^2-x)}}\left[0 + C\right]$$

$$= C e^{-(x^2-x)} = C e^{x-x^2}$$

Using $y = 2$, $x = 1$

$$2 = C e^{1-1^2} = C e^0 = C$$

$$\Rightarrow \boxed{y = 2e^{x-x^2}}$$

34. $x^2 y' - 4xy = 10$ (Dividing through by x^2)

$$\Rightarrow y' - \frac{4xy}{x^2} = \frac{10}{x^2}$$

$P(x) = \frac{4x}{x^2} = \frac{4}{x}$ $\qquad Q(x) = \frac{1}{x^2}$

$\therefore$ 1. $\int P(x)\,dx = \int \frac{4}{x}\,dx = 4\ln|x|$

2. $u(x) = e^{\int P(x)\,dx} = e^{4\ln|x|} = |x|^4 = x^4$

3. $\int Q(x)\,u(x)\,dx = \int \left(\frac{1}{x^2}\right) x^4\,dx$

$$= \int x^2\,dx = \frac{x^3}{3}$$

Final Solution :

$$y = \frac{1}{x^4}\left[\frac{x^3}{3} + C\right]$$

$$= \frac{1}{3x} + \frac{C}{x^4}$$

Using $\underline{y = 10 , \quad x = 1.}$

$$10 = \frac{1}{3(1)} + \frac{C}{1^4} = \frac{1}{3} + C$$

$$\Rightarrow \quad C = 10 - \frac{1}{3} = \frac{29}{3}$$

$$\therefore \quad y = \frac{1}{3x} + \frac{29}{3x^4} = \boxed{\frac{x^3 + 29}{3x^4}}$$