HW 13 errata

30: $\frac{dy}{dt} = \frac{1}{2}y^2 - 2y = \frac{1}{2}y(y-4)$

$$\Rightarrow \int \frac{dy}{y(y-4)} = \int \frac{1}{2}\,dt$$

Use partial fractions on $\frac{1}{y(y-4)}$

$$\frac{1}{y(y-4)} = \frac{A}{y} + \frac{B}{y-4}$$

$$= \frac{A(y-4) + By}{y(y-4)}$$

$$= \frac{(A+B)y - 4A}{y(y-4)}$$

$\therefore \quad A+B=0, \quad -4A=1$

$\Rightarrow A = -\frac{1}{4}, \quad B = \frac{1}{4}.$

$$\therefore \int \frac{dy}{y(y-4)} = \int \left(\frac{-1/4}{y} + \frac{1/4}{y-4}\right) dy$$

$$= -\frac{1}{4}\ln|y| + \frac{1}{4}\ln|y-4|$$

$$= +\frac{1}{4}\left(\ln|y-4| - \ln|y|\right)$$

$$= \frac{1}{4}\ln\left(\frac{|y-4|}{|y|}\right)$$

$$\therefore \int \frac{dy}{y(y-4)} = \int \frac{1}{2}\,dt \quad \text{becomes :}$$

$$\frac{1}{4}\ln\left(\frac{|y-4|}{|y|}\right) = \frac{1}{2}t + C$$

$$\Rightarrow \ln\left(\frac{|y-4|}{|y|}\right) = 2t + 4C$$

Exponentiating both sides,

$$\Rightarrow e^{\ln\left(\frac{|y-4|}{|y|}\right)} = e^{(2t+4C)}$$

$$\Rightarrow \frac{|y-4|}{|y|} = e^{4C}\cdot e^{2t}$$

$$\Rightarrow \frac{y-4}{y} = \pm e^{4C}e^{2t} = C_1 e^{2t} \quad \left(\text{Renaming } \pm e^{4C} = C_1\right)$$

$$\therefore y - 4 = yC_1e^{2t}$$

$$\therefore \quad y - yC_1e^{2t} = 4$$

$$\Rightarrow \quad y(1 - C_1e^{2t}) = 4$$

$$\Rightarrow \quad y = \frac{4}{1 - C_1e^{2t}}$$

Using $y = -3$ when $t = 0$.

$$-3 = \frac{4}{1 - C_1e^{2(0)}} = \frac{4}{1 - C_1}$$

$$\Rightarrow \quad 1 - C_1 = -\frac{4}{3}$$

$$\Rightarrow \quad C_1 = 1 + \frac{4}{3} = \frac{7}{3}.$$

$$\therefore \quad y = \frac{4}{1 - \frac{7}{3}e^{2t}}$$

$$= \frac{12}{3 - 7e^{2t}}$$