HW 11 Solutions MAT 17B

1. $\int_1^2 x^2\,dx$ Midpt. Rule with $n=4$.

Subintervals $[1, 1.25]$, $[1.25, 1.5]$, $[1.5, 1.75]$, $[1.75, 2]$.

Midpts: $\frac{1+1.25}{2} = 1.125$

$\frac{1.25+1.5}{2} = 1.375$

$\frac{1.5+1.75}{2} = 1.625$

$\frac{1.75+2}{2} = 1.875$

Approximation $= (0.25)\times(1.125)^2$
$+ (0.25)\times(1.375)^2$
$+ (0.25)\times(1.625)^2$
$+ (0.25)\times(1.875)^2$

3. $\int_0^1 e^{-x}\,dx$, $n=3$, midpoint Rule.

Subintervals : $\left[0, \frac{1}{3}\right]$, $\left[\frac{1}{3}, \frac{2}{3}\right]$, $\left[\frac{2}{3}, 1\right]$.

Midpts. $\frac{0+\frac{1}{3}}{2} = \frac{1}{6}$

$\frac{\frac{1}{3}+\frac{2}{3}}{2} = \frac{1}{2}$

$\frac{\frac{2}{3}+1}{2} = \frac{5}{6}$

Approximation: $\left(\frac{1}{3}\right)\times\left(e^{-1/6}\right) + \left(\frac{1}{3}\right)\times\left(e^{-1/2}\right) + \left(\frac{1}{3}\right)\times\left(e^{-5/6}\right)$

4. $\int_0^{\pi/2} \sin x\,dx$, $n=4$, Midpt. Rule.

Subintervals : $\left[0, \frac{\pi}{8}\right]$, $\left[\frac{\pi}{8}, \frac{\pi}{4}\right]$, $\left[\pi/4, \frac{3\pi}{8}\right]$

$\left[\frac{3\pi}{8}, \frac{\pi}{2}\right]$

Midpoints: $\frac{0+\pi/8}{2} = \pi/16$

$\frac{\pi/8+\pi/4}{2} = \frac{3\pi}{16}$

$$\frac{\frac{\pi}{4} + \frac{3\pi}{8}}{2} = \frac{5\pi}{16}$$

$$\frac{\frac{3\pi}{8} + \frac{\pi}{2}}{2} = \frac{7\pi}{16}$$

Approximation $= \left(\frac{\pi}{8}\right) \times \sin\left(\frac{\pi}{16}\right) + \left(\frac{\pi}{8}\right) \times \left(\sin \frac{3\pi}{16}\right) + \frac{\pi}{8} \times \sin\left(\frac{5\pi}{16}\right)$

$+ \frac{\pi}{8} \times \left(\sin \frac{7\pi}{16}\right)$

b. $\int_{-1}^{1} (e^{2x} - 1)\,dx$, $n = 4$, Midpt Rule.

Subintervals : $[-1, -0.5]$, $[-0.5, 0]$, $[0, 0.5]$

$[0.5, 1]$.

Midpts : -0.75, -0.25, 0.25, 0.75

Approximation $= (0.5)\left(e^{2(-0.75)} - 1\right)$

$+ (0.5)\left(e^{2(-0.25)} - 1\right)$

$+ (0.5)\left(e^{2(0.25)} - 1\right)$

$+ (0.5)\left(e^{2(0.75)} - 1\right)$

$= \underline{1.48}$

Exact Value $= \int_{-1}^{1} (e^{2x} - 1)\, dx$

$$= \frac{1}{2}e^{2x} - x \Big|_{-1}^{1}$$

$$= \left(\frac{1}{2}e^{2} - 1\right) - \left(\frac{1}{2}e^{-2} + 1\right)$$

$$= \frac{e^{2} - e^{-2}}{2} - 2$$

$$= 1.626$$

9. $\int_{1}^{2} x^2\, dx$, $n = 4$, Trapezoid Rule.

Subintervals $[1, 1.25, 1.5, 1.75, 2]$.

Approximation $= \left(\frac{b-a}{n}\right)\left(\frac{f(x_0)}{2} + f(x_1) + f(x_2) + f(x_3) + \frac{f(x_4)}{2}\right)$

$$= \left(\frac{2-1}{4}\right)\left(\frac{1^2}{2} + (1.25)^2 + (1.5)^2 + (1.75)^2 + \frac{2^2}{2}\right)$$

$$= \frac{1}{4}\left(1 + (1.25)^2 + (1.5)^2 + (1.75)^2 + 2\right)$$

11. $\int_0^1 e^{-x}\,dx$, $n=3$, Trapezoidal Rule.

Subintervals $\left[0, \frac{1}{3}, \frac{2}{3}, 1\right]$

$$\text{Approximation} = \left(\frac{1-0}{3}\right)\left(\frac{e^{-0}}{2} + e^{-1/3} + e^{-2/3} + \frac{e^{-1}}{2}\right)$$

12. $\int_0^{\pi/2} \sin x\,dx$, $n=4$, Trapezoidal Rule.

Subintervals $\left[0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}\right]$

$$\text{Approximation} = \left(\frac{\frac{\pi}{2}-0}{4}\right)\left(\frac{\sin(0)}{2} + \sin\left(\frac{\pi}{8}\right) + \sin\left(\frac{\pi}{4}\right) + \sin\left(\frac{3\pi}{8}\right) + \frac{\sin(\pi/2)}{2}\right)$$

15. $\int_0^2 \sqrt{x}\,dx$, $n=4$, Trapezoidal Rule.

Subintervals $\left[0, \frac{1}{2}, 1, \frac{3}{2}, 2\right]$

$$\text{Approximation} = \left(\frac{2-0}{4}\right)\left(\frac{\sqrt{0}}{2} + \sqrt{\frac{1}{2}} + \sqrt{1} + \sqrt{\frac{3}{2}} + \frac{\sqrt{2}}{2}\right)$$

$= \boxed{1.819}$

Exact Value : $\int_0^2 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_0^2$

$= \frac{2}{3}2^{3/2} - \frac{2}{3}0^{3/2}$

$= \frac{2}{3}\cdot 2^{3/2} = \underline{1.886}$