Homework Problems : WEEK III

Problems to be handed in on Monday, April 29 : Hand in any three of the following problems.

- **1.6.9 (State the identity that holds between** $\sup(A)$ **and** $inf(B)$ **when** r **is negative and prove it),**
- **5 (Prove only the identity involving the supremum),**
- **6 (Prove only the inequality involving the infimum; also state and justify if the inequality can be strict),**
- **11, 12.**

In any exercise, you can use any exercise preceding it and any result stated in class without proof.

1. Let $E \subseteq \mathbb{R}$ be a nonempty set of real numbers that is bounded. Let $-E = \{-x : x \in E\}$. Show that $\inf(-E) = -\sup(E)$, and $\sup(-E) = -\inf(E)$.
2. Let $A \subseteq \mathbb{R}$ be a bounded set of real numbers, and let $r \in \mathbb{R}$ be a fixed real number. Let $B = \{x + r : x \in A\}$. Show that $\sup(A) + r = \sup(B)$ and $\inf(A) + r = \inf(B)$.
3. Do 1.6.9.
4. Let $A \subseteq B$ be two subsets of real numbers. Show that $\inf(A) \geq \inf(B)$ and $\sup(A) \leq \sup(B)$. Is it possible for these inequalities to be strict ?
5. Let A, B be two subsets of real numbers. Let $C = A \cup B$. Show that $\inf(C) = \min\{\inf(A), \inf(B)\}$ and $\sup(C) = \max\{\sup(A), \sup(B)\}$.
6. Let A, B be two subsets of real numbers. Let $C = A \cap B$. Show that $\inf(C) \geq \max\{\inf(A), \inf(B)\}$ and $\sup(C) \leq \min\{\sup(A), \sup(B)\}$. (You can use any of the previous exercises) Is it possible for these inequalities to be strict ?
7. Do 1.6.13 and 1.6.14. The relations will be inequalities - can these inequalities be strict ?
8. Let $E \subseteq \mathbb{R}$ be a set of real numbers. Let L be a lower bound for E, i.e., $L \leq x \quad \forall x \in E$. Show that $L = \inf(E)$ if and only if $\forall \epsilon > 0, \exists x \in E$ such that $x < L + \epsilon$.
9. Do 1.6.23 a), c) and d). Justify your answers.
10. Suppose A, B are two subsets of the real numbers such that $A \cap B \neq \emptyset$. Show that $E = \{|a - b| : a \in A, b \in B\}$ has a minimum element.
11. Consider the function $f(x) = 1 - e^{-x}$ and the set $E = \{f(x) : x \geq 0\}$. Find $\sup(E)$ and justify your answer.
12. Consider the function $f(x) = \frac{1}{x}$ and the set $E = \{f(x) : x > 0\}$. Find $\inf(E)$ and justify your answer.
13. Consider the function $f(x) = \frac{1}{x}$ and the set $E = \{f(x) : x < 0\}$. Find $\sup(E)$ and justify your answer. (Note that x ranges over the *negative* real numbers)
14. Do 1.7.2.