

No notes or calculators. You can leave an answer as a numerical expression without computing the final value. For example, this is a perfectly acceptable answer :

$$((250 - 63)/(1 - e^{(-6 \cdot 3.5)}) * \ln(27/168)). \text{ Show your work clearly !!}$$

1. A bacteria population is observed in a lab which has initial size 3 million. The population grows at a rate proportional to the population size. It becomes double the initial size after $t = 1$ hour.

- (a) (6 points) Write a formula for the population size $S(t)$ at time t as an explicit formula in t . (Your answer should not contain any generic constants).

$$\frac{dS}{dt} = kS \quad \rightarrow \quad \int \frac{1}{S} dS = \int k dt$$

$$\rightarrow \ln|S| = kt$$

$$\rightarrow S = C_1 e^{kt}$$

$$\text{At } t=0, S(0)=3 \rightarrow 3 = C_1$$

$$\text{At } t=1, S(1)=6 \rightarrow 6 = 3e^{k(1)}$$

$$\rightarrow 2 = e^k \rightarrow k = \underline{\ln 2}$$

$$S(t) = 3 e^{(\ln 2)t}$$

- (b) (4 points) How much time does it take for the population to grow to 3 times the initial size.

$$9 = 3 e^{(\ln 2)t}$$

$$\rightarrow 3 = e^{(\ln 2)t}$$

$$\rightarrow \ln 3 = (\ln 2)t$$

$$\rightarrow \boxed{\frac{\ln 3}{\ln 2} = t}$$