

No notes or calculators. You can leave an answer as a numerical expression without computing the final value. For example, this is a perfectly acceptable answer :

$((250 - 63)/(1 - e^{(-6 \cdot 3.5)})) * \ln(27/168)$. Show your work clearly !!

1. (4 points) Compute the Taylor polynomial of degree $n = 3$ about $a = 1$ for the function $f(x) = \ln(x)$.

Taylor polynomial:

$$f(x) = \frac{1}{x}$$

$$f'(x) = -\frac{1}{x^2}$$

$$f''(x) = \frac{2}{x^3}$$

$$= \ln(1) + \frac{1}{1!} \left(\frac{1}{1} \right) (x-1) + \frac{1}{2!} \left(\frac{-1}{1^2} \right) (x-1)^2 + \frac{1}{3!} \left(\frac{2}{1^3} \right) (x-1)^3$$

$$= \boxed{(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}}$$

2. (6 points) Compute the integral.

$$\int_1^e \frac{dx}{x \sqrt{\ln(x)}}$$

$x=1$ is a "problem point" and no other "problem points" in $[1, e]$.

$$\int_1^e \frac{dx}{x \sqrt{\ln x}}$$

Need $\int \frac{dx}{x \sqrt{\ln x}}$

1. $u = \ln x$ 2. $\frac{du}{dx} = \frac{1}{x}$
 $\Rightarrow du = \frac{1}{x} dx$

$$2(\ln e)^{\frac{1}{2}} - 2(\ln 1)^{\frac{1}{2}}$$

3. $\int \frac{1}{\sqrt{u}} du = \frac{u^{\frac{1}{2}}}{\frac{1}{2}} = 2(\ln x)^{\frac{1}{2}}$

$$\lim_{z \rightarrow 1} \left[2(\ln e)^{\frac{1}{2}} - 2(\ln z)^{\frac{1}{2}} \right] = \boxed{2}$$