1. Compute the following integrals (note that some of them are indefinite integrals and some are definite integrals).

(a) **(4 pts)** $\int (2x+2)sec^2(x^2+2x-5)dx$.

1. $u = x^2 + 2x - 5$
2. $\frac{du}{dx} = 2x + 2. \Rightarrow du = (2x+2)\,dx$
3. $\int (2x+2)\sec^2(x^2+2x-5)dx = \int \sec^2 u\,du = \tan u$
4. $\int (2x+2)\sec^2(x^2+2x-5)dx = \boxed{\tan(x^2+2x-5)+C}$

(b) **(4 pts)** $\int \frac{(x+5)^2}{x^{1/3}}dx$.

$$= \int \frac{x^2+25+10x}{x^{1/3}}dx = \int x^{2-1/3} + 25x^{-1/3} + 10x^{1-1/3}\,dx$$

$$= \int x^{5/3} + 25x^{-1/3} + 10x^{2/3}\,dx$$

$$= \boxed{\frac{x^{8/3}}{8/3} + \frac{25x^{2/3}}{2/3} + \frac{10x^{5/3}}{5/3} + C}$$

(c) **(5 pts)** $\int_0^1 x(1-x)^{10}dx$.

1. $\boxed{u = 1-x} \to \boxed{x = 1-u}$
2. $\frac{du}{dx} = -1 \Rightarrow \boxed{-du = dx}$
3. $\int x(1-x)^{10}dx = \int (1-u)u^{10}(-du) = -\int (u^{10} - u^{11})du$

$$= -\frac{u^{11}}{11} + \frac{u^{12}}{12}$$

4. $\int_0^1 x(1-x)^{10}dx = \left[-\frac{(1-x)^{11}}{11} + \frac{(1-x)^{12}}{12}\right]_0^1$

$$\int_0^1 x(1-x)^{10}dx$$

$$= \left(\frac{0}{11} + \frac{0}{11}\right) - \left(\frac{-(1)^{11}}{11} + \frac{1^{12}}{12}\right)$$

$$= \boxed{\frac{1}{11} - \frac{1}{12}}$$

(d) **(6 pts)** $\int x sec^2(x)dx$.

Use integration by parts:

$f(x) = x \qquad g(x) = sec^2 x$.

1. $\int g(x)dx = G(x) = \int sec^2 x\, dx = \tan x = \underline{G(x)}$.

2. $\int f'(x)\, G(x) = \int (1) \tan x\, dx = \int \tan x\, dx = \ln|\cos x|$

3. $\int f(x)g(x) = f(x)G(x) - \int f'(x)G(x) = \boxed{x\tan x - \ln|\cos x| + C}$

(e) **(7 pts)** $\int \frac{4x+1}{x^2-3x-10}dx$.

No need for long division.

Factorize denominator: $\frac{4x+1}{(x-5)(x+2)}$

Use partial fractions:

$$\frac{4x+1}{(x-5)(x+2)} = \frac{A}{x-5} + \frac{B}{x+2}.$$

$$= \frac{A(x+2) + B(x-5)}{(x-5)(x+2)}$$

$$= \frac{(A+B)x + (2A-5B)}{(x-5)(x+2)}$$

So $\int \frac{4x+1}{(x+5)(x+2)}dx = \int \frac{3}{x-5}dx + \int \frac{1}{x+2}dx$

$= \boxed{3\ln|x-5| + \ln|x+2| + C}$

$A+B = 4 \qquad 2A-5B = 1$

$A = 4-B$

$\Rightarrow 2(4-B) - 5B = 1$

$\Rightarrow 8 - 7B = 1$

$\Rightarrow \boxed{B = 1}$

$\boxed{A = 3}$

(f) **(7 pts)** $\int_1^4 \ln(\sqrt{x})dx$.

1. $u = \sqrt{x}$ 2. $\frac{du}{dx} = \frac{1}{2\sqrt{x}} \to 2\sqrt{x}\,du = dx$

$\Rightarrow 2u\,du = dx$

$\Rightarrow \int \ln\sqrt{x}\,dx = \int \ln(u)(2u\,du) = 2\int u(\ln u)du$

Use integration by parts for $\int u \ln u\, du$.

$f(u) = \ln u \qquad g(u) = u.$

1. $\int g(u)du = \frac{u^2}{2} = G(u).$

2. $\int f'(u)G(u)\,du = \int \left(\frac{1}{u}\right)\frac{u^2}{2}du = \int \frac{u}{2}du = \frac{u^2}{4}$

$\Rightarrow \int f(u)g(u) = (\ln u)\frac{u^2}{2} - \frac{u^2}{4}$

$\therefore 2\int u \ln u = 2\left((\ln u)\frac{u^2}{2} - \frac{u^2}{4}\right) = (\ln u)u^2 - \frac{u^2}{2}$

$\therefore \int_1^4 \ln\sqrt{x}\,dx = \left(\ln\sqrt{x}\right)(\sqrt{x})^2 - \frac{(\sqrt{x})^2}{2}\Big|_1^4 = \left((\ln\sqrt{x})x - \frac{x}{2}\right)\Big|_1^4$

$= \left((\ln\sqrt{4})4 - \frac{4}{2}\right) - \left((\ln\sqrt{1})1 - \frac{1}{2}\right)$

$= \boxed{4\ln 2 - 2 + \frac{1}{2}}$

(g) **(5 pts)** $\int tan^2(x)dx$. (Hint : You might need to use a trigonometric identity)

$\tan^2 x = \sec^2 x - 1$

$\therefore \int \tan^2 x\,dx = \int (\sec^2 x - 1)\,dx$

$= \int \sec^2 x\,dx - \int 1\,dx$

$= \boxed{\tan x - x + C}$

(h) **(7 pts)** $\int \frac{e^{-x}-1}{xe^{-x}-1}dx$. (Hint: Multiply the numerator and denominator by e^x)

$$\int \frac{(e^{-x}-1)(e^x)}{(xe^{-x}-1)(e^x)}\,dx = \int \frac{e^{-x}e^x - e^x}{xe^{-x}e^x - e^x}\,dx$$

$$= \int \frac{e^0 - e^x}{xe^0 - e^x}\,dx = \int \frac{1-e^x}{x-e^x}\,dx$$

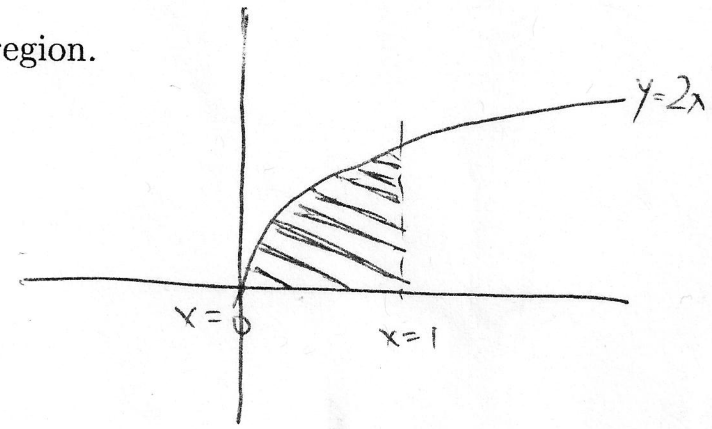
1. $u = x - e^x$ 2. $\frac{du}{dx} = 1 - e^x \Rightarrow du = (1-e^x)dx$

3. $\int \frac{1-e^x}{x-e^x}dx = \int \frac{1}{u}\,du = \ln|u|$

4. $\int \frac{1-e^x}{x-e^x}dx = \boxed{\ln|x-e^{-x}| + C}$

2. Consider the region bounded by the graphs of $y = 2x^{1/3}$, $y = 0$, $x = 0$ and $x = 1$.

(a) **(9 pts)** Compute the AREA of this region.



$$\int_0^1 (2x^{1/3})dx$$

$$= \frac{2x^{4/3}}{4/3}\Big|_0^1$$

$$= \frac{2(1)^{4/3}}{4/3} - \frac{2(0)^{4/3}}{4/3} = \boxed{\frac{3}{2}}$$

(b) **(6 pts)** Set up (but DO NOT EVALUATE) the integral to compute the volume of the solid obtained by revolving this region around the x-axis.

$$\int_0^1 \pi \left(2x^{1/3}\right)^2 dx$$

$$= \int_0^1 \pi\, 4x^{2/3}\, dx = \int_0^1 4\pi\, x^{2/3}\, dx.$$

All of these are correct answers

3. **(10 pts)** Approximate the following integral using the sum of areas of rectangles after dividing the interval of interest into smaller subintervals.

$$\int_{-1}^{3} \frac{1}{x^2+1}\,dx$$

Use 4 equal subintervals and the left endpoints for the function values.

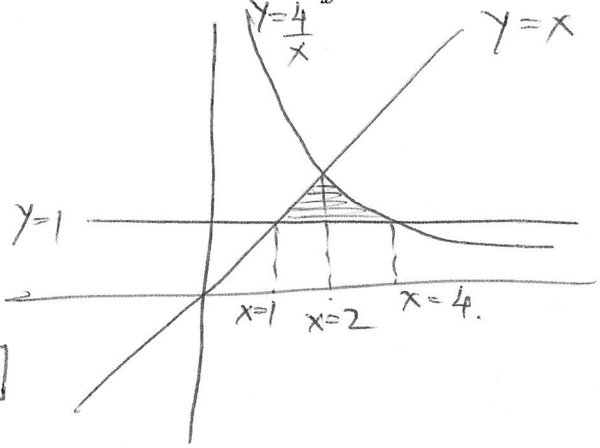
$-1 \quad 0 \quad 1 \quad 2 \quad 3$

$$\int_{-1}^{3} \frac{1}{x^2+1}\,dx = \boxed{\left(\frac{1}{(-1)^2+1}\right)(1) + \left(\frac{1}{0^2+1}\right)(1) + \left(\frac{1}{1^2+1}\right)(1) + \left(\frac{1}{2^2+1}\right)(1)}$$

$$= \boxed{\frac{1}{2} + 1 + \frac{1}{2} + \frac{1}{5}}$$

both are OK

4. **(7 pts)** Set up (but DO NOT EVALUATE) the definite integral to compute the area of the region bounded by the graphs of $y = x$, $y = \frac{4}{x}$ and $y = 1$.



Find 3 pts of intersection :

For $y=x$ and $y=\frac{4}{x}$ $\Rightarrow$ $x=\frac{4}{x}$ $\Rightarrow$ $x^2=4$

$\Rightarrow$ $\boxed{x=+2}$

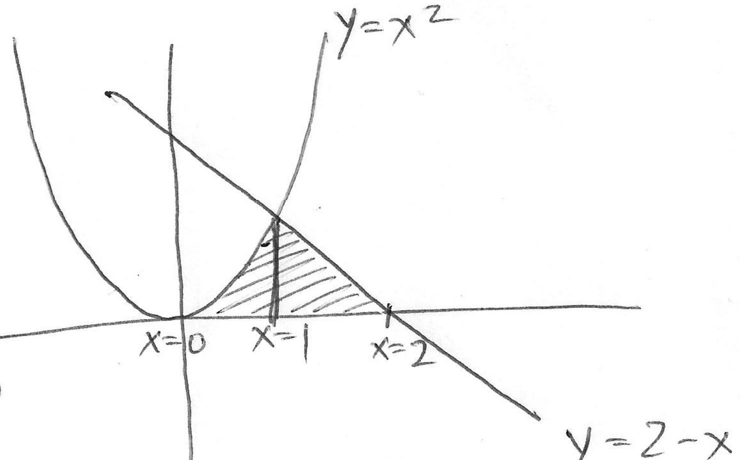
For $y=x$ and $y=1$ $\Rightarrow$ $\boxed{x=1}$

For $y=\frac{4}{x}$ and $y=1$ $\Rightarrow$ $\frac{4}{x}=1$ $\Rightarrow$ $\boxed{x=4}$

Area splits into 2 regions :

$\therefore$ Area $= \boxed{\int_1^2 (x-1)\,dx + \int_2^4 \left(\frac{4}{x}-1\right)dx}$

5. **(8 pts)** Consider the region bounded by the graphs of $y = x^2$, $y = 2 - x$, and $y = 0$. Set up (but DO NOT EVALUATE) the integral(s) to compute the volume of the solid obtained by revolving this region about the x axis.



Find pt. of intersection for splitting the area into 2 regions :

$y=x^2$ and $y=2-x$.

$\Rightarrow$ $x^2=2-x$ $\Rightarrow$ $x^2+x-2=0$

$\Rightarrow$ $(x+2)(x-1)=0$

$\Rightarrow$ $\boxed{x=1}$, -2

$\therefore$ Area $= \boxed{\int_0^1 \pi(x^2)^2\,dx + \int_1^2 \pi(2-x)^2\,dx}$