1. Compute the following integrals (note that some of them are indefinite integrals and some are definite integrals).

(a) **(4 pts)** $\int (x+1)e^{(x^2+2x-5)}dx$.

1. $u = x^2+2x-5$

2. $\frac{du}{dx} = 2x+2 \implies \frac{1}{2}du = (x+1)dx$

3. $\int e^{x^2+2x-5}(x+1)dx = \int e^u \frac{1}{2}du = \frac{1}{2}e^u$

$= \boxed{\frac{1}{2}e^{x^2+2x-5} + C}$

(b) **(4 pts)** $\int \frac{x^3+5x}{\sqrt{x}}dx$.

$= \int \frac{x^3}{\sqrt{x}} + \frac{5x}{\sqrt{x}}\, dx = \int x^{5/2} + 5x^{1/2}\, dx$

$= \boxed{\frac{x^{7/2}}{7/2} + \frac{5x^{3/2}}{3/2} + C}$

(c) **(5 pts)** $\int_0^1 x(1-x)^{1/3}dx$.

1. $u = 1-x$

2. $\frac{du}{dx} = -1 \implies -du = dx$

3. $\int (1-u)u^{1/3}(-du) = \int -u^{1/3} + u^{4/3}\, du$

$= \frac{u^{7/3}}{7/3} - \frac{u^{4/3}}{4/3}$

$= \left.\frac{(1-x)^{7/3}}{7/3} - \frac{(1-x)^{4/3}}{4/3}\right|_0^1$

$= \boxed{\left(\frac{0}{7/3} - \frac{0}{4/3}\right) - \left(\frac{1}{(7/3)} - \frac{1}{4/3}\right)}$

(d) **(6 pts)** $\int x sin(x)dx$.

Using Integration by Parts: $f(x) = x$, $g(x) = \sin(x)$

1. $\int g(x)dx = \int \sin(x) = -\cos(x). = G(x)$

2. $\int f'(x)\, G(x)dx = \int (1)(-\cos(x))\, dx = -\int \cos(x)dx = -\sin x$

3. $\int x \sin x \, dx = x(-\cos(x)) - (-\sin x)$

$$= \boxed{\sin x - x\cos(x) + C}$$

(e) **(7 pts)** $\int \frac{2x-1}{x^2+5x+6}dx$.

Proper Rational Function:

Factorize denominator: $x^2+5x+6 = (x+2)(x+3)$

$$\frac{2x-1}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3} = \frac{A(x+3) + B(x+2)}{(x+2)(x+3)}$$

$$= \frac{(A+B)x + (3A+2B)}{(x+2)(x+3)}.$$

$$\rightarrow \left.\begin{array}{l} A+B = 2 \\ 3A+2B = -1 \end{array}\right\} \Rightarrow A = 2-B$$

$$\Downarrow$$

$$3(2-B) + 2B = -1$$

$$\Rightarrow 7 = B$$

$$\Rightarrow A = -5$$

$$\therefore \int \frac{2x-1}{x^2+5x+6}dx = \int \frac{-5}{x+2} + \frac{7}{x+3}dx = \boxed{-5\ln|x+2| + 7\ln|x+3| + C}$$

(f) **(7 pts)** $\int_1^4 x^3 e^{x^2} dx$.

1. $u = x^2$ 2. $\frac{du}{dx} = 2x \implies \frac{1}{2}du = x\,dx$.

3. $\int x^3 e^{x^2} dx = \int x^2 e^{x^2} x\,dx = \int u e^u \cdot \frac{1}{2} du$.

$= \frac{1}{2} \int u e^u du$.

Use Integration by Parts:

$\int u e^u du \qquad f(u) = u,\ g(u) = e^u$

1. $\int g(u) du = \int e^u du = e^u = G(u)$.
2. $\int f'(u) G(u) du = \int (1) e^u du = e^u$.
3. $\int u e^u du = u e^u - e^u$

$\therefore \int_1^4 x^3 e^{x^2} dx = \frac{1}{2}\left(x^2 e^{x^2} - e^{x^2}\right)\Big|_1^4 = \boxed{\frac{1}{2}\left[\left(4^2 e^{4^2} - e^{4^2}\right) - \left(1e^1 - e^1\right)\right]}$

(g) **(5 pts)** $\int sec^4(x)dx$. (Hint : Use $sec^2(x) = 1 + tan^2(x)$)

$\int \sec^4(x) dx = \int \sec^2 x \sec^2 x\, dx$

$= \int (1 + \tan^2 x) \sec^2 x\, dx$

1. $u = \tan x$.
2. $\frac{du}{dx} = \sec^2 x \implies du = \sec^2 x\, dx$.
3. $\int (1 + u^2) du = u + \frac{u^3}{3} = \boxed{\tan x + \frac{\tan^3 x}{3} + C}$

(h) **(7 pts)** $\int e^{2x}\sqrt{1+e^x}\,dx$.

$$= \int e^x \sqrt{1+e^x}\; e^x\,dx$$

1. $u = 1+e^x$ 2. $\frac{du}{dx} = e^x$ $\to du = e^x dx$

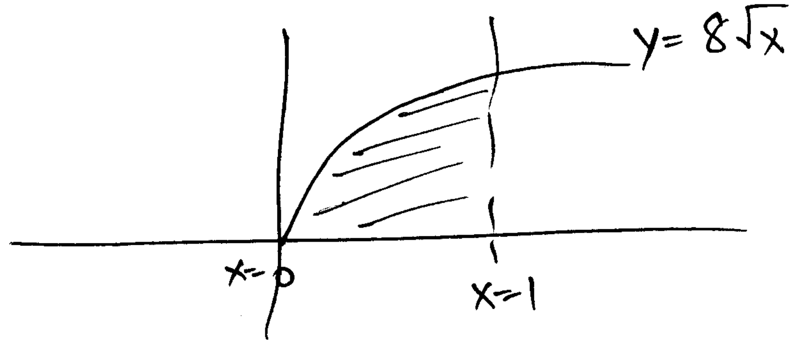
3. $\int \sqrt{u}\,(u-1)\,du = \int u^{3/2} - u^{1/2}\,du$

$$= \frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}$$

$$= \boxed{\frac{(1+e^x)^{5/2}}{5/2} - \frac{(1+e^x)^{3/2}}{3/2} + C}$$

2. Consider the region bounded by the graphs of $y = 8\sqrt{x}$, $y = 0$, and $x = 1$.

(a) **(9 pts)** Compute the AREA of this region.



$$\int_0^1 (8\sqrt{x} - 0)\,dx = \int_0^1 8\sqrt{x}\,dx = \left.\frac{8x^{3/2}}{3/2}\right|_0^1$$

$$= \boxed{8\left(\frac{1}{3/2} - \frac{0}{3/2}\right)} = \boxed{\frac{16}{3}}$$

both are OK.

(b) **(6 pts)** Set up (but DO NOT EVALUATE) the integral to compute the volume of the solid obtained by revolving this region around the x-axis.

$$\boxed{\pi \int_0^1 (8\sqrt{x})^2 - 0^2 \, dx} = \boxed{\pi \int_0^1 16x \, dx}$$

both are OK.

3. **(10 pts)** The temperature of an ice cream sandwich as a function of time t ($t = 0$ being when it is removed from the freezer) is given by

$$T(t) = \frac{60t}{1 + 4t^2}$$

for $0 \le t \le 1/2$. What is the average temperature over this time period?

$$\text{Avg. temperature} = \frac{\int_0^{1/2} \frac{60t}{1+4t^2}\,dt}{\frac{1}{2} - 0}$$

Compute $\int \frac{60t}{1+4t^2}\,dt$:

1. $u = 1+4t^2$ 2. $\frac{du}{dt} = 8t$

3. $\int \frac{60t}{1+4t^2}\,dt = \int \frac{60}{1+4t^2}\, t\,dt$

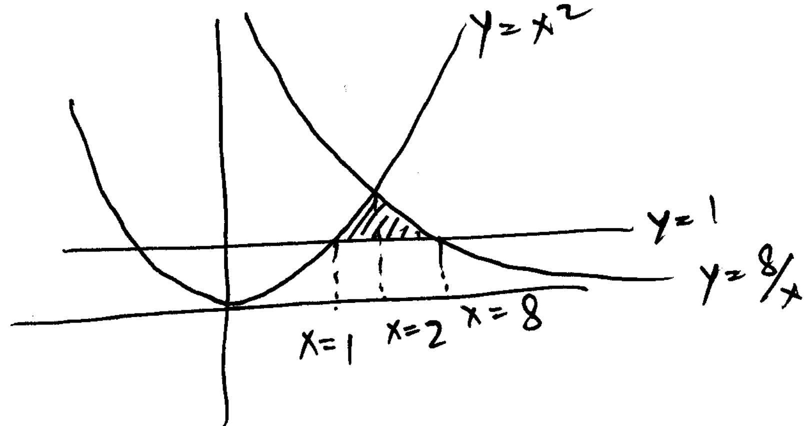
$$= \int \frac{60}{u}\,\frac{1\,du}{8} = \frac{60}{8}\ln|u|$$

$$\therefore \frac{\int_0^{1/2} \frac{60t}{1+4t^2}\,dt}{\frac{1}{2}-0} = \frac{\frac{60}{8}\ln|1+4t^2|\Big|_0^{1/2}}{\frac{1}{2}} = \frac{\frac{60}{8}\left(\ln\left|1+4\left(\frac{1}{4}\right)\right| - \ln|1|\right)}{\frac{1}{2}}$$

4. **(7 pts)** Set up (but DO NOT EVALUATE) the definite integral to compute the area of the region bounded by the graphs of $y = x^2$, $y = \frac{8}{x}$ and $y = 1$.

Pts. of intersection:

1. $y = 1$, $y = x^2 \Rightarrow x = 1$
2. $y = x^2$, $y = \frac{8}{x} \Rightarrow x = 2$
3. $y = 1$, $y = \frac{8}{x} \Rightarrow x = 8.$



$$\text{Area} = \boxed{\int_1^2 (x^2 - 1)\,dx + \int_2^8 \left(\frac{8}{x} - 1\right) dx.}$$

5. **(8 pts)** Consider the region bounded by the graphs of $y = x$, $y = (x-2)^2$, and $y = 0$. Set up (but DO NOT EVALUATE) the integral(s) to compute the volume of the solid obtained by revolving this region about the x axis.

Pt. of intersection:

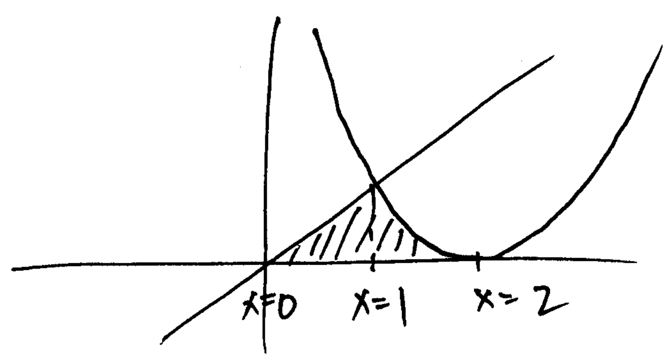
$y = x \quad y = (x-2)^2$

$x = x^2 - 4x + 4$

$0 = x^2 - 5x + 4$

$= (x-4)(x-1)$

$x = 1, 4$



$$\text{Volume} = \boxed{\pi \int_0^1 \left((x)^2 - 0^2\right) dx + \pi \int_1^2 \left(\left((x-2)^2\right)^2 - 0^2\right) d}$$

$$= \boxed{\pi \int_0^1 x^2\,dx + \pi \int_1^2 (x-2)^4\,dx}$$ — both are OK.