Section 6.3.3

16. Average value of $g(t) = \sin(\pi t)$ over $[-1,1]$

$$= \frac{\int_{-1}^{1} \sin(\pi t)\,dt}{1-(-1)}$$

$$= \frac{-\frac{1}{\pi}\cos(\pi(1)) - \left(-\frac{1}{\pi}\cos(\pi(-1))\right)}{2}$$

$$= \frac{1}{2\pi}\left(\cos(-\pi) - \cos(\pi)\right) = \frac{1}{2\pi}\left(-1-(-1)\right) = \boxed{0}$$

27 b) Average value of $T(t) = 68 + \sin\left(\frac{\pi}{12}t\right)$ over $0 \le t \le 24$.

$$= \frac{\int_0^{24}\left(68 + \sin\left(\frac{\pi}{12}t\right)\right)dt}{24-0}$$

Antiderivative of $68 + \sin\left(\frac{\pi}{12}t\right)$ is $68t - \frac{12}{\pi}\cos\left(\frac{\pi}{12}t\right)$

$\therefore$ Avg. value $= \dfrac{\left(68\times 24 - \frac{12}{\pi}\cos\left(\frac{\pi}{12}\times 24\right)\right) - \left(68\cdot 0 - \frac{12}{\pi}\cos\left(\frac{\pi}{12}\times 0\right)\right)}{24}$

$$= \frac{68 \times 24 - \frac{12}{\pi}\cos 2\pi + \frac{12}{\pi}\cos(0)}{24}$$

$$= \frac{68 \times 24 - \frac{12}{\pi} \times 1 + \frac{12}{\pi} \times 1}{24}$$

$$= 68$$

32 b) Avg. velocity

$$= \frac{\int_0^6 \left(-(t-3)^2 + 5\right) dt}{6-0}$$

Antiderivative of $-(t-3)^2 + 5 = -t^2 - 9 + 6t + 5$

$$= -t^2 + 6t - 4$$

is $-\frac{t^3}{3} + \frac{6t^2}{2} - 4t$

$\therefore$ Avg velocity $= \dfrac{\left(-\frac{6^3}{3} + \frac{6 \cdot 6^2}{2} - 4 \cdot 6\right) - \left(\frac{0^3}{3} + \frac{6 \cdot 0^2}{2} - 4 \cdot 0\right)}{6}$

$$= \frac{12}{6} = 2$$

32c) We want t^* s.t $v(t^*)$ is the average velocity $= 2$

$$\therefore \quad -(t^*-3)^2 + 5 = 2$$

$$\Rightarrow \quad 3 = (t^*-3)^2$$

$$\Rightarrow \quad \sqrt{3} = t^* - 3$$

$$\Rightarrow \quad \boxed{3+\sqrt{3} = t^*.}$$

Section 6.3.4

36. Rotating $\sqrt{2x}$ about x-axis :

$$\text{Volume} = \int_0^2 \pi(\sqrt{2x})^2\,dx = \int_0^2 \pi\,(2x^2)\,dx$$

$$= 2\pi \int_0^2 x^2\,dx$$

$$= 2\pi\left(\frac{2^3}{3} - \frac{0^3}{3}\right)$$

$$= \boxed{\frac{16\pi}{3}}$$

38. ~~Rot~~ Rotating $y = e^x$ over the interval $x=0$ to $x = \ln 2$ around the x-axis.

$$\text{Volume} = \int_0^{\ln 2} \pi (e^x)^2 \, dx$$

$$= \int_0^{\ln 2} \pi e^{2x} \, dx = \pi\left(\frac{1}{2} e^{2(\ln 2)} - \frac{1}{2} e^{2\cdot 0}\right)$$

$$= \pi\left(\frac{1}{2}(4) - \frac{1}{2}\right) = \frac{3\pi}{2}$$

39. Rotating $y = \sec x$ around x-axis over $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$.

$$\text{Volume} = \int_{-\pi/3}^{\pi/3} \pi (\sec x)^2 \, dx = \int_{-\pi/3}^{\pi/3} \pi \sec^2 x \, dx$$

$$= \pi\left(\tan \frac{\pi}{3} - \tan\left(-\frac{\pi}{3}\right)\right)$$

$$= \boxed{2\pi\sqrt{3}}$$

40. Rotating $y = \sqrt{1-x^2}$ around x-axis over $[0,1]$.

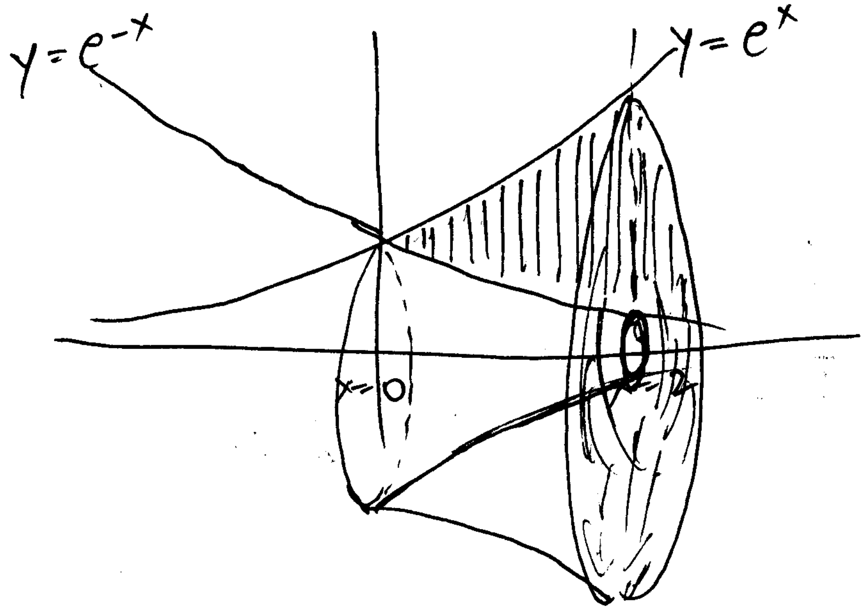
$$\text{Volume} = \int_0^1 \pi\left(\sqrt{1-x^2}\right)^2 dx$$

$$= \int_0^1 \pi(1-x^2)\,dx = \pi\left(\left(1-\frac{1^3}{3}\right)-\left(0-\frac{0^3}{3}\right)\right)$$

$$= \frac{2\pi}{3}$$

43.

$y = e^x \qquad y = e^{-x} \qquad 0 \le x \le 2$



Volume $= \pi \int_0^2 (e^x)^2 - (e^{-x})^2 \, dx$

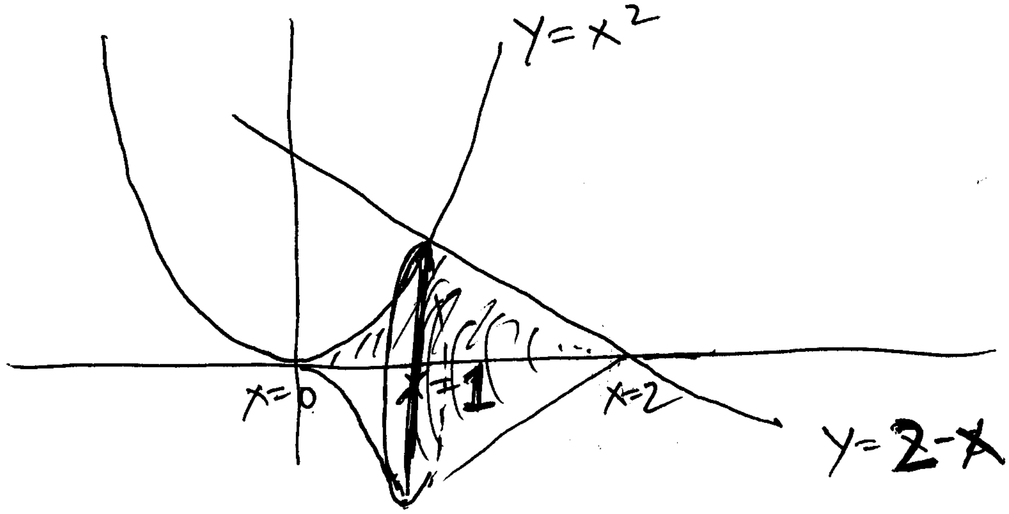
$= \pi \int_0^2 e^{2x} - e^{-2x} \, dx$

$= \pi \left[\frac{1}{2} e^{2x} - \left(\frac{1}{-2}\right) e^{-2x} \right] \Big|_0^2$

$= \pi \left[\left(\frac{1}{2} e^4 + \frac{1}{2} e^{-4} \right) - \left(\frac{1}{2} e^0 + \frac{1}{2} e^0 \right) \right]$

$= \boxed{\pi \left(\frac{e^4 + e^{-4}}{2} - 1 \right)}$

Problem 6 Volume version



Need to split volume into two like we did for area:

$$\pi \int_0^1 (x^2)^2 \, dx + \pi \int_1^2 (2-x)^2 \, dx$$

$$= \pi \frac{x^5}{5} \Big|_0^1 + \pi \int_1^2 (4 - 4x + x^2) \, dx$$

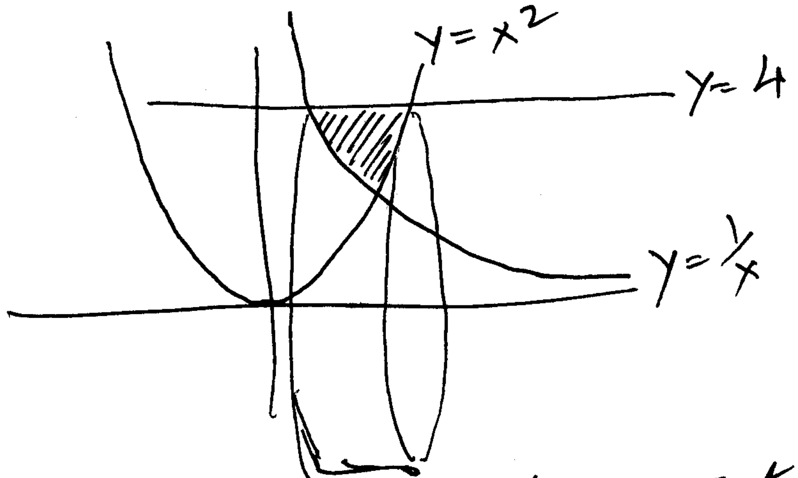
$$= \pi \frac{x^5}{5} \Big|_0^1 + \pi \left(4x - 2x^2 + \frac{x^3}{3}\right) \Big|_1^2$$

$$= \pi \left(\frac{1}{5}\right) + \pi \left[\left(4(2) - 2(2)^2 + \frac{2^3}{3}\right) - \left(4(1) - 2(1)^2 + \frac{1^3}{3}\right) \right]$$

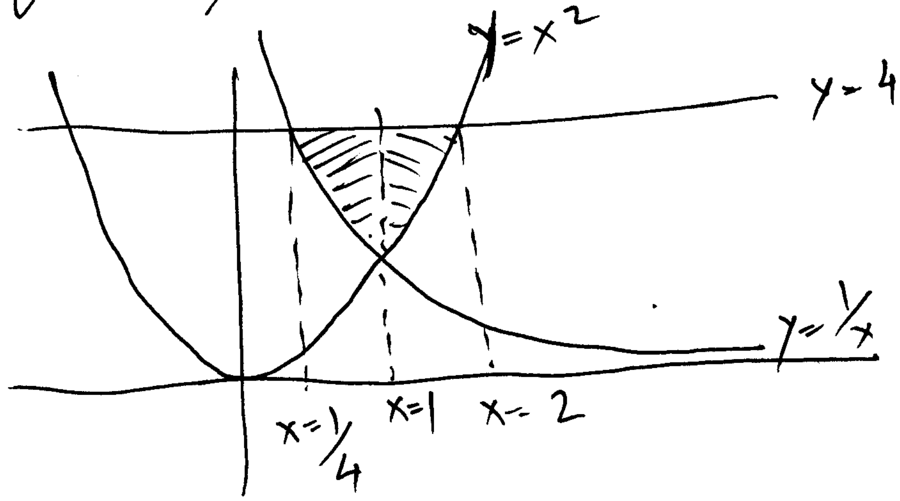
$$= \frac{\pi}{5} + \pi \left[\frac{8}{3} - 2 - \frac{1}{3} \right] = \frac{\pi}{5} + \frac{\pi}{3}$$

$$= 8\pi/15$$

Problem 7 Volume version.



Again split volume into 2 :



$$\pi\int_{1/4}^{1}\left(4^2-\left(\frac{1}{x}\right)^2\right)dx \quad + \quad \pi\int_{1}^{2} 4^2-(x^2)^2\ dx$$

$$= \pi\int_{1/4}^{1}\left(16-\frac{1}{x^2}\right)dx + \pi\int_{1}^{2}\left(16-x^4\right)dx$$

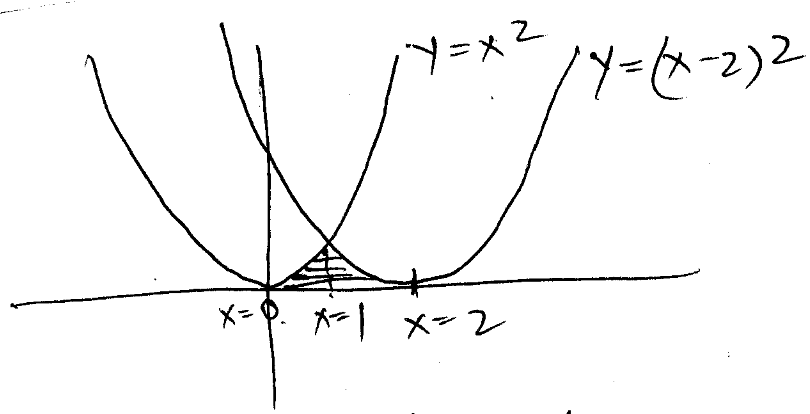
$$= \pi\left(16x-\left(-\frac{1}{x}\right)\right)\Bigg|_{1/4}^{1} + \pi\left[16x-\frac{x^5}{5}\right]\Bigg|_{1}^{2}$$

$$= \pi\left[\left(16 + \frac{1}{1}\right) - \left(16\left(\frac{1}{4}\right) + \frac{1}{(1/4)}\right)\right]$$

$$+ \pi\left[\left(16(2) - \frac{2^5}{5}\right) - \left(16(1) - \frac{1}{5}\right)\right]$$

$$= 9\pi + \frac{49}{5}\pi$$

Problem 10 **Volume version**



Find split point : $x^2 = (x-2)^2$

$$x^2 = x^2 - 4x + 4$$

$$\Rightarrow 4x = 4 \Rightarrow x = 1$$

$$\text{Volume} = \pi\int_0^1 (x^2)^2\, dx + \pi\int_1^2 ((x-2)^2)^2 dx$$

$$= \pi\int_0^1 x^4\, dx + \pi\int_1^2 (x^2 - 4x + 4)^2 dx$$

$$= \pi \frac{x^5}{5}\Big|_0^1 + \pi \int_1^2 (x^2-4x+4)(x^2-4x+4)\,dx$$

$$= \pi \frac{x^5}{5}\Big|_0^1 + \pi \int_1^2 (x^4-4x^3+4x^2-4x^3+16x^2-16x$$
$$+4x^2-16x+16)\,dx$$

$$= \pi \frac{x^5}{5}\Big|_0^1 + \pi \int_1^2 (x^4-8x^3+24x^2-32x+16)\,dx$$

$$= \frac{\pi}{5} + \pi\left[\frac{x^5}{5} - \frac{8x^4}{4} + \frac{24x^3}{3} - \frac{32x^2}{2} + 16x\right]_1^2$$

$$= \frac{\pi}{5} + \pi\left[\left(\frac{2^5}{5} - \frac{8(2)^4}{4} + \frac{24(2)^3}{3} - \frac{32(2)^2}{2} + 16(2)\right)\right.$$
$$\left. - \left(\frac{1}{5} - \frac{8}{4} + \frac{24}{3} - \frac{32}{2} + 16\right)\right].$$