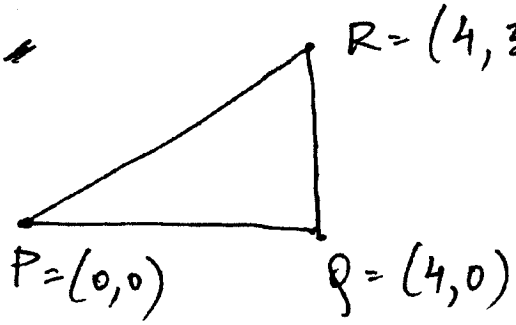
HW 24 Solutions MAT 17B

31 b)



Length of side PQ

= length of vector Q − P

= " " " $\begin{bmatrix}4\\0\end{bmatrix} - \begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix}4\\0\end{bmatrix}$

$= \sqrt{4^2 + 0^2} = (4)$

Length of side QR

= length of vector R − Q

= " " " $\begin{bmatrix}4\\3\end{bmatrix} - \begin{bmatrix}4\\0\end{bmatrix} = \begin{bmatrix}0\\3\end{bmatrix}$

$= \sqrt{0^2 + 3^2} = (3)$

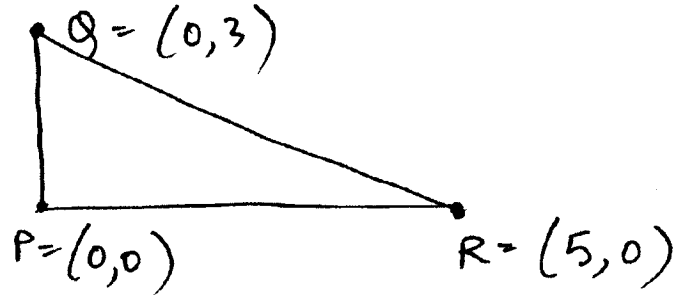
Length of side PR

= length of vector R − P

= " " " $\begin{bmatrix}4\\3\end{bmatrix} - \begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix}4\\3\end{bmatrix}$

$= \sqrt{4^2 + 3^2} = \sqrt{25} = (5)$

32. b)



Length of side PR = length of vector R − P

$$= \text{" " "} \quad \begin{bmatrix}5\\0\end{bmatrix} - \begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix}5\\0\end{bmatrix}$$

$$= \sqrt{5^2 + 0^2} = 5$$

Length of side QR = length of vector Q − R.

$$= \text{" " "} \quad \begin{bmatrix}0\\3\end{bmatrix} - \begin{bmatrix}5\\0\end{bmatrix} = \begin{bmatrix}-5\\3\end{bmatrix}$$

$$= \sqrt{(-5)^2 + 3^2} = \sqrt{34}$$

Length of side PQ = length of vector Q − P

$$= \text{" " "} \quad \begin{bmatrix}0\\3\end{bmatrix} - \begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix}0\\3\end{bmatrix}$$

$$= \sqrt{0^2 + 3^2} = 3$$

Angle at corner P = angle between R − P and Q − P.

$$= \text{angle between } \begin{bmatrix}5\\0\end{bmatrix} \text{ and } \begin{bmatrix}0\\3\end{bmatrix}$$

$$= \arccos\left(\frac{\begin{bmatrix}5\\0\end{bmatrix} \cdot \begin{bmatrix}0\\3\end{bmatrix}}{5 \times 3}\right) = \arccos(0) = \frac{\pi}{2}$$

Angle at corner R = angle between P−R and Q−R.

$$= \text{“ “ } \begin{bmatrix} -5 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

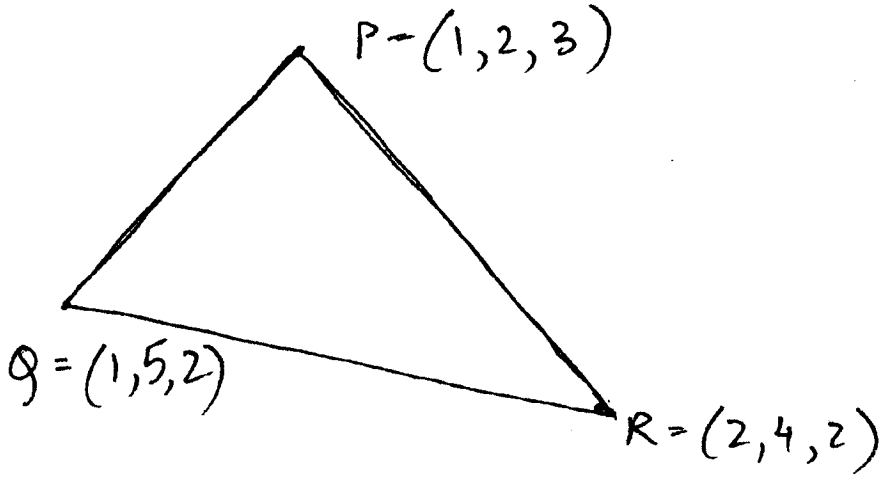
$$= \arccos\left(\frac{\begin{bmatrix} -5 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -5 \\ 3 \end{bmatrix}}{\sqrt{(-5)^2+0^2}\,\sqrt{(-5)^2+3^2}}\right) = \arccos\left(\frac{25}{5\times\sqrt{34}}\right)$$

Angle at corner Q = angle between P−Q + R−Q.

$$= \text{“ “ } \begin{bmatrix} 0 \\ -3 \end{bmatrix} + \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

$$= \arccos\left(\frac{\begin{bmatrix} 0 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -3 \end{bmatrix}}{\sqrt{0^2+(-3)^2}\,\sqrt{5^2+(-3)^2}}\right) = \arccos\left(\frac{9}{3\times\sqrt{34}}\right)$$

33.



a) Length of side QR = length of vector R−Q.

$$= \text{" } \text{"} \quad \begin{bmatrix}2\\4\\2\end{bmatrix} - \begin{bmatrix}1\\5\\2\end{bmatrix} = \begin{bmatrix}1\\-1\\0\end{bmatrix}$$

$$= \sqrt{1^2+(-1)^2+0^2} = \boxed{\sqrt{2}}$$

Length of side PR = length of vector P − R

$$= \text{" } \text{"} \quad \begin{bmatrix}1\\2\\3\end{bmatrix} - \begin{bmatrix}2\\4\\2\end{bmatrix} = \begin{bmatrix}-1\\-2\\1\end{bmatrix}$$

$$= \sqrt{(-1)^2+(-2)^2+1^2} = \boxed{\sqrt{6}}$$

Length of side QP = length of vector P − Q

$$= \text{" } \text{"} \quad \begin{bmatrix}1\\2\\3\end{bmatrix} - \begin{bmatrix}1\\5\\2\end{bmatrix} = \begin{bmatrix}0\\-3\\1\end{bmatrix}$$

$$= \sqrt{0^2+(-3)^2+1^2} = \boxed{\sqrt{10}}$$

b) Angle at corner Q = angle between vectors $R-Q$ and $P-Q$.

= angle between $\begin{bmatrix}1\\-1\\0\end{bmatrix}$ and $\begin{bmatrix}0\\-3\\1\end{bmatrix}$

$$= \arccos\left(\frac{\begin{bmatrix}1\\-1\\0\end{bmatrix}\cdot\begin{bmatrix}0\\-3\\1\end{bmatrix}}{\sqrt{2}\ \sqrt{10}}\right) = \arccos\left(\frac{3}{\sqrt{20}}\right)$$

Angle at corner P = angle between $Q-P$ and $R-P$

= " between $\begin{bmatrix}0\\3\\-1\end{bmatrix}$ and $\begin{bmatrix}1\\2\\-1\end{bmatrix}$

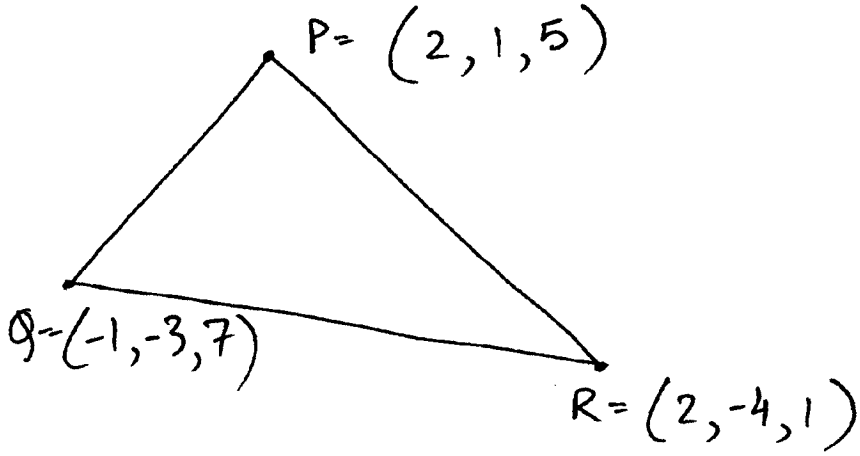
$$= \arccos\left(\frac{\begin{bmatrix}0\\3\\-1\end{bmatrix}\cdot\begin{bmatrix}1\\2\\-1\end{bmatrix}}{\sqrt{10}\ \sqrt{6}}\right) = \arccos\left(\frac{7}{\sqrt{60}}\right)$$

Angle at corner R = angle between $Q-R$ + $P-R$.

= " " $\begin{bmatrix}-1\\1\\0\end{bmatrix}$ + $\begin{bmatrix}-1\\-2\\1\end{bmatrix}$

$$= \arccos\left(\frac{\begin{bmatrix}-1\\1\\0\end{bmatrix}\cdot\begin{bmatrix}-1\\-2\\1\end{bmatrix}}{\sqrt{2}\ \sqrt{6}}\right) = \arccos\left(\frac{-1}{\sqrt{12}}\right)$$

34.



a) Length of side QR = length of vector $R - Q$.

$$= \text{" } \text{"} \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix} - \begin{bmatrix} -1 \\ -3 \\ 7 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ -6 \end{bmatrix}$$

$$= \sqrt{3^2 + (-1)^2 + (-6)^2} = \sqrt{46}$$

Length of side PQ = length of vector $Q - P$

$$= \text{" } \text{"} \begin{bmatrix} -3 \\ -4 \\ 2 \end{bmatrix}$$

$$= \sqrt{(-3)^2 + (-4)^2 + 2^2} = \sqrt{29}$$

Length of side PR = length of vector $R - P$

$$= \text{" } \text{"} \begin{bmatrix} 0 \\ -5 \\ -4 \end{bmatrix}$$

$$= \sqrt{0^2 + (-5)^2 + (-4)^2} = \sqrt{41}$$

b) Angle at corner Q = angle between R-Q and P-Q.

$$= \text{"} \quad \text{"} \begin{bmatrix} 3 \\ -1 \\ -6 \end{bmatrix} \text{ and } \begin{bmatrix} 3 \\ 4 \\ -2 \end{bmatrix}$$

$$= \arccos\left(\frac{\begin{bmatrix} 3 \\ -1 \\ -6 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \\ -2 \end{bmatrix}}{\sqrt{3^2+(-1)^2+(-6)^2}\,\sqrt{3^2+4^2+(-2)^2}}\right) = \arccos\left(\frac{17}{\sqrt{46}\,\sqrt{29}}\right)$$

Angle at corner P = angle between Q-P and R-P.

$$= \text{"} \quad \text{"} \begin{bmatrix} -3 \\ -4 \\ 2 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ -5 \\ -4 \end{bmatrix}$$

$$= \arccos\left(\frac{\begin{bmatrix} -3 \\ -4 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -5 \\ -4 \end{bmatrix}}{\sqrt{(-3)^2+(-4)^2+2^2}\,\sqrt{0^2+(-5)^2+(-4)^2}}\right) = \arccos\left(\frac{12}{\sqrt{29}\,\sqrt{41}}\right)$$

Angle at corner R = angle between Q-R and P-R.

$$= \text{"} \quad \text{"} \begin{bmatrix} -3 \\ 1 \\ 6 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 5 \\ 4 \end{bmatrix}$$

$$= \arccos\left(\frac{\begin{bmatrix} -3 \\ 1 \\ 6 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 5 \\ 4 \end{bmatrix}}{\sqrt{(-3)^2+1^2+6^2}\,\sqrt{0^2+5^2+4^2}}\right) = \arccos\left(\frac{29}{\sqrt{46}\,\sqrt{41}}\right)$$