

52.

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$$

1. New matrix for eigenvalues:

$$\begin{bmatrix} -1-\lambda & 0 \\ 0 & 2-\lambda \end{bmatrix}$$

2. set determinant to 0.

$$(-1-\lambda)(2-\lambda) = 0.$$

$$\Rightarrow (\lambda-2)(\lambda+1) = 0.$$

$$\Rightarrow \lambda = 2, -1.$$

3. Using  $\lambda = 2$ :

$$\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 2 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

$$-x_1 = 2x_1$$

$$+2x_2 = 2x_2$$

$$\Rightarrow x_1 = 0$$

$x_2$  can be anything

$$x_1 = 0, x_2 = 1$$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$  is an eigenvector with eigenvalue 2.

Using  $\lambda = -1$ .

$$\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$-x_1 = -x_1$$

$$2x_2 = x_2$$

$\Rightarrow x_2 = 0$ ,  $x_1$  can be anything.

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$  is an eigenvector with eigenvalue -1.

53.  $\begin{bmatrix} -4 & 2 \\ -3 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} -4-\lambda & 2 \\ -3 & 1-\lambda \end{bmatrix}$

set Determinant to 0.

$$(-4-\lambda)(1-\lambda) - 2(-3) = 0.$$

$$\lambda^2 + 3\lambda + 2 = 0$$

$$(\lambda+2)(\lambda+1) = 0$$

$$\Rightarrow \lambda = -2, -1.$$

Using  $\lambda = -2$ :

$$\begin{bmatrix} -4 & 2 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -2 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{aligned} -4x_1 + 2x_2 &= -2x_1 & \Rightarrow 2x_2 &= 2x_1 \\ -3x_1 + x_2 &= -2x_2 & \Rightarrow x_1 &= x_2 \end{aligned}$$

choose  $x_1 = 1, x_2 = 1$ .

so,  $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$  eigenvector with eigenvalue -2

Using  $\lambda = -1$ :

$$\begin{bmatrix} -4 & 2 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$-4x_1 + 2x_2 = -x_1 \Rightarrow x_2 = \frac{3x_1}{2}$$

$$-3x_1 + x_2 = -x_2$$

Pick  $x_1 = 1, x_2 = \frac{3}{2}$

$\begin{bmatrix} 1 \\ 1.5 \end{bmatrix}$  is an eigenvector with eigenvalue -1

54.  $\begin{bmatrix} 3 & 6 \\ -1 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 3-\lambda & 6 \\ -1 & -4-\lambda \end{bmatrix}$

so  $(3-\lambda)(-4-\lambda) + 6 = 0$

$\Rightarrow \lambda^2 + \lambda - 6 = 0$

$\Rightarrow (\lambda+3)(\lambda-2) = 0$

$\Rightarrow \lambda = 2, -3.$

Using  $\lambda = -3$ :

$$\begin{bmatrix} 3 & 6 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -3 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$3x_1 + 6x_2 = -3x_1 \Rightarrow 6x_2 = -6x_1$$

$$-x_1 - 4x_2 = -3x_2 \Rightarrow x_1 = -x_2$$

choose  $x_1 = 1, x_2 = -1$

$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$  is an eigenvector with eigenvalue  $-3$ .

Using  $\lambda = 2$ :  $\begin{bmatrix} 3 & 6 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 2 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$3x_1 + 6x_2 = 2x_1$$

$$6x_2 = -x_1$$

$$-x_1 - 4x_2 = 2x_2$$

choose  $x_1 = 6, x_2 = -1$ .

$\begin{bmatrix} 6 \\ -1 \end{bmatrix}$  is an eigenvector with eigenvalue  $2$ .

55.  $\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 2-\lambda & 1 \\ 2 & 3-\lambda \end{bmatrix}$

$$\Rightarrow (2-\lambda)(3-\lambda) - (1)(2) = 0$$

$$\Rightarrow \lambda^2 - 5\lambda + 4 = 0$$

$$(\lambda-4)(\lambda-1) = 0$$

$$\lambda = 4, 1.$$

Using  $\lambda=4$ :  $\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 4 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\Rightarrow 2x_1 + x_2 = 4x_1 \quad \Rightarrow x_2 = 2x_1$$

$$2x_1 + 3x_2 = 4x_2$$

$$\Rightarrow \text{choose } x_1 = 1, x_2 = 2$$

$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$  is an eigenvector with eigenvalue 4

Using  $\lambda=1$ :  $\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\Rightarrow 2x_1 + x_2 = x_1 \quad \Rightarrow x_2 = -x_1$$

$$2x_1 + 3x_2 = x_2$$

$$\text{choose } x_1 = 1, x_2 = -1$$

$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$  is an eigenvector with eigenvalue 1.

57.  $\begin{bmatrix} 4-\lambda & 0 \\ 0 & 3-\lambda \end{bmatrix} \Rightarrow (4-\lambda)(3-\lambda) = 0$   
 $\Rightarrow \boxed{\lambda = 3, 4}$

58.  $\begin{bmatrix} -7-\lambda & 0 \\ 0 & 6-\lambda \end{bmatrix} \Rightarrow (-7-\lambda)(6-\lambda) = 0$   
 $\Rightarrow \boxed{\lambda = -7, 6}$

59.  $\begin{bmatrix} 1-\lambda & -3 \\ 0 & 2-\lambda \end{bmatrix} \Rightarrow (1-\lambda)(2-\lambda) = 0$   
 $\Rightarrow \boxed{\lambda = 1, 2}$

60.  $\begin{bmatrix} -1-\lambda & 4 \\ 0 & -2-\lambda \end{bmatrix} \Rightarrow (-1-\lambda)(-2-\lambda) = 0$   
 $\Rightarrow \boxed{\lambda = -1, -2}$