ON THE DIFFERENCE BETWEEN MEDIAN AND MEAN OF GAMMA DISTRIBUTIONS AND A RELATED RAMANUJAN SEQUENCE

Abstract

Let $Y_\mu$ be Poisson distributed with expectation $\mu$ and let $\lambda_n$ be determined by $P(Y_{\lambda_n} \leq n) = 1/2$. Then $\lambda_n$ is also the median of the distribution $\Gamma(n + 1, 1)$.

What can be said about $\lambda_n$, or about $\alpha_n = \lambda_n - n$?

This problem is closely related to the following problems by Ramanujan:

1. Show that

$$\frac{e^n}{2} = 1 + n + \frac{n^2}{2} + \cdots + \frac{n^{n-1}}{(n-1)!} + \theta_n \cdot \frac{n^n}{n!},$$

where $1/3 < \theta_n \leq 1/2$ for $n = 0, 1, 2, \ldots$.

2. Show that

$$\theta_n = \frac{1}{3} + \frac{4}{135(n + k_n)},$$

where $2/21 < k_n \leq 8/45$. 