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SEMINAR

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Preseminar: 3:00 p.m.
Refreshments: 3:30 p.m.
Seminar: 4:00 p.m.

ISOTONIC REGRESSION:  
ANOTHER LOOK AT THE CHANGE POINT PROBLEM

Abstract

In simple versions of the change point problem, independent random variables $X_k$ have one (marginal) distribution $F_0$, say, for $k < \nu$, and another $F_1$ for $k \geq \nu$, where the change point $\nu$ is an unknown parameter and $1 \leq \nu \leq \infty$. Statistical questions include testing for the existence of a change, and estimating the location of $\nu$ when it exists. The problem arises in industrial quality assessment but also more generally—for example, in assessing changes in weather patterns and disease rates. In this talk, I will explore a modified version of the change point problem in which the abrupt change is replaced by a monotonic, but otherwise arbitrary, sequence of changes. In the simplest case, suppose that $X_k = \mu_k + \epsilon_k$, where $\mu_1, \ldots, \mu_n$ are the unknown parameters and $\epsilon_1, \ldots, \epsilon_n$ are independent normal random variables with a common variance $\sigma^2$. Suppose that $\mu_1 \leq \cdots \leq \mu_n$ and consider testing the hypothesis $H_0 : \mu_1 = \cdots = \mu_n$. This is the same null hypothesis encountered in the change point problem, but the alternative is different. A penalized likelihood ratio test of $H_0$ is developed and its asymptotic distribution is obtained. The asymptotic distribution is obtained under the more general assumption that $\epsilon_1, \ldots, \epsilon_n$ are part of a zero-mean, square-integrable, stationary ergodic process that exhibits suitable short-range dependence. The test is illustrated by rainfall data from the Tucuman Region of Argentina.