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Numerical Analysis 550.681
Homework #6
04.04.2008

Kincaid 6.2: 4,5,9,17
6.3: 1,2

Kincaid 6.2.4 Prove that if f is a polynomial of degree k, then for $n > k$,

$$f[x_0, x_1, \ldots, x_n] = 0$$

$$f[x_0, x_1, \ldots, x_n] = \frac{f^{(n)}(\xi)}{n!} \qquad \text{(for some } \xi \in (a,b)\text{, Theorem 6.2.4)}$$

$$f^{(n)} = 0 \qquad \text{(since degree } k < n\text{)}$$

$$\Rightarrow \quad f[x_0, x_1, \ldots, x_n] = 0$$

∎

Kincaid 6.2.5 Prove that if p is a polynomial of degree at most n, then

$$p(x) = \sum_{i=0}^{n} p[x_0, x_1, \ldots, x_i] \prod_{j=0}^{i-1} (x - x_j)$$

The coefficients of the Newton interpolating polynomial are exactly the divided differences. For $(x_i, p(x_i)),\ i = 0, 1, \ldots, n$, this is

$$\sum_{i=0}^{n} p[x_0, x_1, \ldots, x_i] \prod_{j=0}^{i-1} (x - x_j)$$

We also know that p is a polynomial that interpolates these same points. Since this polynomial is unique,

$$p(x) = \sum_{i=0}^{n} p[x_0, x_1, \ldots, x_i] \prod_{j=0}^{i-1} (x - x_j)$$

∎

Kincaid 6.2.9 Prove this formula:

$$f[x_0, x_1, \ldots, x_n] = \sum_{i=0}^{n} f(x_i) \prod_{\substack{j=0 \\ j \neq i}}^{i-1} (x_i - x_j)^{-1}$$

We can compare the coefficient of x^n in the following two formulas.

$$\sum_{i=0}^{n} f[x_0, x_1, \ldots, x_i] \prod_{j=0}^{i-1}(x - x_j) \Rightarrow f[x_0, x_1, \ldots, x_n]x^n + \ldots \tag{9.1}$$

$$\sum_{i=0}^{n} f(x_i) l_i(x) \Rightarrow [\sum_{i=0}^{n} f(x_i) \prod_{\substack{j=0 \\ j\neq i}}^{n} (x_i - x_j)^{-1}]x^n + \ldots \tag{9.2}$$

Since (9.1) = (9.2), we get

$$\Rightarrow \qquad f[x_0, x_1, \ldots, x_n] = \sum_{i=0}^{n} f(x_i) \prod_{\substack{j=0 \\ j\neq i}}^{i-1} (x_i - x_j)^{-1}$$

■

Kincaid 6.2.17 Find the Newton interpolating polynomial for these data:

x	1	3/2	0	2
$f(x)$	3	13/4	3	5/3

x	$f[x]$	$f[x,x]$	$f[x,x,x]$	$f[x,x,x,x]$
1	3	1/2	1/3	−2
3/2	13/4	1/6	−5/3	
0	3	−2/3		
2	5/3			

$$\Rightarrow \quad p(x) = 3 + \frac{1}{2}(x-1) + \frac{1}{3}(x-1)(x-\frac{3}{2}) - 2x(x-1)(x-\frac{3}{2})$$

■

Kincaid 6.3.1 Use the extended Newton divided difference method to obtain a quartic polynomial that takes these values:

x	0	1	2
$p(x)$	2	−4	44
$p'(x)$	−9	4	

x	$f[x]$	$f[x,x]$	$f[x,x,x]$	$f[x,x,x,x]$	$f[x,x,x,x,x]$
0	2				
0	2	−9			
1	−4	−6	3		
1	−4	4	10	7	
2	44	48	44	17	5

$$\Rightarrow \quad p(x) = 2 - 9x + 3x^2 + 7x^2(x-1) + 5x^2(x-1)^2$$

■

Kincaid 6.3.2 Find the quartic polynomial that takes the values given in the preceding problem and, in addition, satisfies $p(3) = 2$. Hint: Add a suitable term to the polynomial found in the preceding problem.

$$\begin{aligned} p(x) &= 2 - 9x + 3x^2 + 7x^2(x-1) + 5x^2(x-1)^2 + cx^2(x-1)^2(x-2) \\ p(3) &= 2 = 36c + 308 \\ &\Rightarrow c = -8.5 \end{aligned}$$

$$\Rightarrow \quad p(x) = 2 - 9x + 3x^2 + 7x^2(x-1) + 5x^2(x-1)^2 - 8.5x^2(x-1)^2(x-2)$$

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