

Problem 1. Find the general solution to $xy'(x) - y(x) = 1$.

Problem 2. Solve $\frac{dy}{dx} = -(x - y)^2$, $y(0) = 0$.

Problem 3. Consider the differential equation $(4x - y) dx + (6y - x) dy = 0$.

(a) Show this equation is exact.

(b) Solve it.

Problem 4. Label each of the following statements as either True or False.

(a) For any two row-equivalent $n \times n$ matrices \mathbf{A} and \mathbf{B} , $\det(\mathbf{A}) = \det(\mathbf{B})$.

(b) If $\det(\mathbf{A}) \neq 0$, then the linear system $\mathbf{Ax} = \mathbf{b}$ is consistent for every choice of vector \mathbf{b} .

(c) For every symmetric $n \times n$ matrix \mathbf{A} the reduced echelon form of \mathbf{A} is also symmetric.

(d) If \mathbf{E}_1 and \mathbf{E}_2 are elementary matrices, then their product $\mathbf{E}_1\mathbf{E}_2$ is an elementary matrix.

(e) If \mathbf{E}_1 and \mathbf{E}_2 are elementary matrices, then their product $\mathbf{E}_1\mathbf{E}_2$ is row-equivalent to the Identity.

Problem 5. Consider the 4×4 matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 4 \end{bmatrix}.$$

- (a) Compute $\det(\mathbf{A})$.
- (b) Find $\mathbf{A}_{4,3}$. (Hint: This is the cofactor of some entry.)
- (c) Compute \mathbf{A}^{-1} .

Problem 6. Consider the following linear system:

$$\begin{aligned}x_1 - 2x_2 + x_3 + 2x_4 &= b_1 \\2x_1 - 4x_2 + 2x_3 + x_4 &= b_2 \\x_1 - 2x_2 + x_3 &= b_3\end{aligned}$$

The corresponding augmented coefficient matrix of this linear system in reduced echelon form is given by

$$\begin{bmatrix} 1 & -2 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 0 & d_3 \end{bmatrix}.$$

- (a) For which values of d_1, d_2 and d_3 is the corresponding linear system consistent?
- (b) For which values of d_1, d_2 and d_3 is the corresponding linear system inconsistent?
- (c) Identify the leading variables and the free variables. Clearly label each.
- (d) Find the solution set of the corresponding homogeneous linear system.