550.291 Linear Algebra & Differential Equations **Fall 2009**

Homework #5: due in class Friday, Oct. 16

From the text:
Chapter 4.1 / 10, 16, 20
Chapter 4.2 / 4, 5, 28*

*in class I showed that the set of vectors $\mathbf{x}$ such that $\mathbf{A}\mathbf{x} = 0\mathbf{x}$, i.e., the nullspace of $\mathbf{A}$ is a subspace, this problem wants you to show the statement remains true if we change 0 to any constant k.

5.1. Consider the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0 & 1 & -1 \\ 2 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}.$$

This matrix can be thought of as a mapping of $\mathbb{R}^5$ into $\mathbb{R}^3$ in the following way: $\mathbf{A}$ takes the vector $\mathbf{x} \in \mathbb{R}^5$ to the vector $\mathbf{A}\mathbf{x} \in \mathbb{R}^3$. In this case the nullspace of the matrix $\mathbf{A}$ is the subset of $\mathbb{R}^5$ that $\mathbf{A}$ maps to the zero vector in $\mathbb{R}^3$, i.e., $\text{Null}(\mathbf{A}) := \{\mathbf{x} \in \mathbb{R}^5 : \mathbf{A}\mathbf{x} = \mathbf{0}\}$. Another way is to think that Null($\mathbf{A}$) is the set of all solutions to the homogeneous linear system with coefficient matrix $\mathbf{A}$.
Find a basis for Null($\mathbf{A}$)