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SEMINAR

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THE GENERALIZED MEDIAN STABLE MATCHINGS

Abstract

In the *stable marriage problem* (SM), there are n men and n women each of whom has a list that ranks all individuals of the opposite sex. A *matching* is a set of man–woman pairs where each individual appears in at most one pair. The objective of the problem is to find a matching μ that has n pairs and has no *blocking pairs*—i.e., a man and a woman who prefer each other over their partners in μ. A celebrated result by Gale and Shapley in the 1960s states that every instance of SM has a stable matching that can be found in $O(n^2)$ time. Today, centralized stable matching algorithms match medical residents to hospitals, students to schools, etc.

Let I be a stable matching instance with N stable matchings. For each man m, order his (not necessarily distinct) N partners from his most preferred to his least preferred. Denote the ith woman in his sorted list as $p_i(m)$. Let α_i consist of the man–woman pairs where each man m is matched to $p_i(m)$. In the late 1990s, Teo and Sethuraman proved this surprising result: For $i = 1, \ldots, N$, not only is α_i a matching, it is also stable. The α_i's are called the *generalized median stable matchings* of I.

In this talk, we present a new characterization of the generalized median stable matchings that provides interesting insights. It implies that the generalized median stable matchings in the middle—$\alpha_{(N+1)/2}$ when N is odd, $\alpha_{N/2}$ and $\alpha_{(N/2)+1}$ when N is even—are fair not only in a local sense but also in a global sense because they are also medians of the lattice of stable matchings. We then show that there are some families of SM instances for which computing an α_i is easy but that the task is NP-hard in general. Finally, we also consider the generalized median stable matchings for the *stable roommates problem* (SR), a "non-bipartite" version of the stable marriage problem. In particular, we prove that the global structure governing a set of SR stable matchings is a median graph.

(This research is, in part, joint with Anhua Lin.)