CONVEX POLYTOPES WITH ABELIAN, VERTEX-TRANSITIVE SYMMETRY

Abstract

In studying families of mathematical objects, it is often useful to single out those having a lot of symmetry in the hope that, for these, the analysis will be made less difficult by use of the symmetry. For convex polytopes, such a simplifying assumption might be that the polytope has a vertex-transitive group of symmetries. It is curious that such polytopes having more complicated groups of symmetries are often easier to study than those having, say, abelian groups of symmetries. T. Bisztriczky and I consider the problem of classifying the convex polytopes $P$ for which there is an abelian group of orientation-preserving linear symmetries acting transitively on the set of vertices. We consider in particular the case of 4-dimensional polytopes, an investigation fruitfully begun by Z. Smilansky. There is an interesting connection between the combinatorial types of the polytopes and continued fractions.