EXACT RECONSTRUCTION OF SPARSE SIGNALS
FROM VERY FEW MEASUREMENTS

ABSTRACT

In our work we show how through solving a nonconvex minimization problem we can exactly reconstruct signals from very few measurements. This is equivalent to solving an underdetermined system of linear equations which in general has an infinite number of solutions. What helps us find a unique solution is the fact that the signals we are interested in are usually sparse or can have a sparse representation. The importance of this result is that in many settings it is too difficult or too expensive to acquire enough information. Applications can be found in medical and seismic imaging, remote sensing, and sensor networks, among many other disciplines.

The field known as compressed sensing has recently developed a technique for exact signal recovery from fewer measurements than was believed to be possible. The key to success is having a certain relationship between sparsity and number of measurements and solving an $L_1$-norm minimization problem. We can obtain the correct solution with even fewer measurements by minimizing the $L_p$-quasi-norm of the signal, where $0 < p < 1$. So at the price of losing convexity we can reconstruct signals which are unrecoverable by current techniques. We give a geometric and probabilistic interpretation of why this is true.

(This is joint work with Rick Chartrand, Mathematical Modeling and Analysis Group, Los Alamos National Laboratory.)