DOT-PRODUCT REPRESENTATIONS OF GRAPHS:
EXACT, ASYMPTOTIC, PROJECTIVE, ALGORITHMIC,
APPROXIMATE, AND APPLIED

Abstract

Generally speaking, a dot-product representation of a simple graph $G = (V, E)$ is a mapping $X : V \rightarrow \mathbb{R}^d$ so that adjacency of distinct vertices $v$ and $w$ is linked to the dot product $X(v) \cdot X(w)$. The most demanding requirement we place is that $X(v) \cdot X(w)$ be 1 if the vertices are adjacent and be 0 otherwise; this is an exact dot-product representation. Failing that, we may require that the dot products be arbitrarily close to their intended targets, arriving at the notion of asymptotic dot-product representation. And if this is not achievable, we may simply try to get the best possible representation we can. We also consider a variant in which vertices are mapped to points in real projective space and show how this variant is equivalent to the asymptotic case. We show the existence of polynomial-time algorithms for computing representations in fixed dimensions. And we explain why we care about such representations and the role they play in network analysis.

(The work we present was done in collaboration with Kim Tucker.)