HEDGING UNDER $L^2$ CONVEX RISK MEASURES

Abstract

We consider the problem of minimizing the risk of a financial position (hedging) in an incomplete market. It is well known that the industry standard for risk measure, the Value at Risk, does not take into account the natural idea that risk should be minimized through diversification. This observation led to the theory of convex risk measures by Follmer and Schied. But as a theory on bounded financial positions, it is not ideally suited for the problem of hedging because simple buy-hold strategies may not be bounded. Therefore, we propose as an alternative to extend convex risk measures as functionals on $L^2$. This framework is more suitable for optimal hedging with $L^2$-valued financial markets, where the theory of stochastic integration plays a natural role. We then study the problem of hedging financial contracts with optimal trading strategies to minimize the convex risk measure. In the case of constrained trading strategies we also prove the existence of the optimal hedge.