

Department of Applied Mathematics and Statistics  
The Johns Hopkins University

SEMINAR

Daniel Q. Naiman  
Dept. of Applied Mathematics & Statistics  
The Johns Hopkins University

September 14, 2006  
304 Whitehead Hall  
Refreshments: 3:30 p.m.  
Seminar: 4:00 p.m.

ARBITRAGE AND GEOMETRY

ABSTRACT

Arbitrage is a fundamental notion in mathematical finance, and making the “no free lunch” assumption that arbitrage opportunities in the marketplace are unavailable has played a fundamental role in financial economics. The Arbitrage Theorem, an important example of a *theorem of the alternative*, will be explained for the case of an  $m \times n$  payoff matrix corresponding to  $m$  scenarios and  $n$  investments. For an appropriate definition of a *random* payoff matrix, we use geometric reasoning to show that the probability of an arbitrage opportunity is

$$\sum_{i=0}^{n-1} \binom{m-1}{i} / 2^{m-1}.$$

As a corollary, we conclude that if the number of scenarios  $m$  is even and the number of available investments is  $m/2$ , then the probability of an arbitrage opportunity is  $1/2$ .

(This is joint work with Edward R. Scheinerman.)