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SEMINAR

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304 Whitehead Hall  
Refreshments: 3:30 p.m.  
Seminar: 4:00p.m.

MAXIMUM MATCHING ON GRAPHS FOR  
KIDNEY PAIRED DONATION

ABSTRACT

Relatives and friends of an end-stage renal disease patient who offer to donate a kidney are often found to be incompatible with their intended recipients. Kidney paired donation matches one patient and his incompatible donor with another patient and donor in the same situation for an organ exchange. Let the patient-donor pairs be the vertices of a graph  $G$  with an edge between two vertices if a paired donation is possible. The naive optimization problem finds the matching of maximum cardinality in  $G$ . However, some matches are geographically undesirable. Also, the lifespan of a transplanted kidney depends on the immunologic compatibility of the donor and the recipient. Thus it is better to view  $G$  as an edge-weighted graph and to seek a matching with maximum weight. Unfortunately, such matchings might not have the maximum number of edges; there is a risk of an unpredictable trade-off between the quality and the quantity of paired kidney donations.

We propose a method of weighting the edges of  $G$  that guarantees that every matching with maximum weight also has maximum cardinality. Our proposal reduces travel and favors immunologic compatibility without decreasing the total number of kidney paired donations. Our method is based on some elementary inequalities relating edge weights and matching numbers in graphs and is easy to implement using standard optimization techniques.