**Department of Applied Mathematics and Statistics**
**The Johns Hopkins University**

## SEMINAR

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**Friday**, March 30, 2006
304 Whitehead Hall
Seminar: **11:00 a.m.**
in 304 Whitehead Hall
Luncheon reception to follow
in 301 Whitehead Hall

# PROBLEMS IN CONVEXITY

Abstract

Convexity arises in mathematics in many contexts, and in particular in stochastic optimization theory and linear programming. These two are actually the same subject, despite the fact that many people in stochastic optimization (including myself for many years) seem not fully aware of the link between the two subjects, namely, duality theory. One can also view tomography in terms of convexity theory. Additionally, many of the standard inequalities in the book by Hardy, Littlewood, and Polya, which was written before von Neumann introduced linear programming, can be proved by duality theory in convexity. An inequality of Mark Brown (given a new proof recently by Olkin and me) says:

$$\mathcal{F}(X+Y) \leq \mathcal{F}(X) + \mathcal{F}(Y),$$

where $X$ and $Y$ are positive and independent and $\mathcal{F}(Z) := (\mathbf{E}\, Z)/(\mathbf{E}\, Z^2)$.

Here is another cute problem (Taglamitsky's theorem) that can viewed in terms of convexity: If

$$\left| f^{(k)}(x) \right| \leq e^{-x} \text{ for all } x \geq 0, \quad k = 0, 1, \ldots,$$

show that $f(x) \equiv ce^{-x}$ for some $|c| \leq 1$.

The commonality of the various special problems is not fully understood even by the speaker. A brand-new problem is still partly open: This is the problem (formulated by Jack Denny for a problem in medical imaging) of maximizing

$$\mathbf{P}(X+Y > 1)$$

over all independent symmetric random variables $X$ and $Y$ each with variance $\sigma^2$. The audience is encouraged to contribute to the solution as well as to the general understanding of the commonality of such problems. Problem solvers (I'm one) often fail to see the forest for the trees.