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SEMINAR

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September 22, 2005
304 Whitehead Hall
Refreshments: 3:30 p.m.
Seminar: 4:00 p.m.

EXACT AND ASYMPTOTIC DOT PRODUCT REPRESENTATIONS OF GRAPHS

Abstract

Let $G = (V, E)$ be a (finite, simple) graph and let d be a nonnegative integer. An *exact dot product representation in dimension* d of G is a mapping $\mathbf{X} : V \to \mathbb{R}^d$ with the property that for all $u \neq v$ in V we have

$$\mathbf{X}(u) \cdot \mathbf{X}(v) = \begin{cases} 1 & \text{if } uv \in E, \text{ and} \\ 0 & \text{if } uv \notin E. \end{cases}$$

The least d for which G has such a representation is denoted $\mathrm{dp}(G)$.

An *asymptotic dot product representation in dimension* d of G is a sequence of mappings $\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \ldots : V \to \mathbb{R}^d$ with the property that for all $u \neq v$ in V we have

$$\lim_{k\to\infty} \mathbf{X}_k(u) \cdot \mathbf{X}_k(v) = \begin{cases} 1 & \text{if } uv \in E, \text{ and} \\ 0 & \text{if } uv \notin E. \end{cases}$$

The least d for which G has such a representation is denoted $\mathrm{dp}^*(G)$.

In this talk we motivate these definitions (based on an "inverse" problem in the study of random dot product graphs) and present a variety of results. For example, if G is a bipartite graph, then $\mathrm{dp}^*(G)$ equals the maximum size of a matching in G.

(This is joint work with Kimberly Tucker.)