RANDOM WALKS: SIMPLE AND SELF-AVOIDING

Abstract

I will discuss a number of problems concerning the set of points visited by a “random walker” on the $d$-dimensional integer lattice. The kinds of questions we will ask are:

- What is the “dimension” of the set of points visited by the walker?
- How does this change if we forbid the walker to visit points it has already visited?
- If two different walkers start at different points and one walker paints all vertices it visits red and the other one paints all vertices it visits blue, what is the chance that no vertex ends up purple?

Many of these questions are motivated by critical phenomena in statistical physics. The answers depend very heavily on the dimension $d$ in which the walker moves. In some cases the answers are well known, other questions have been answered recently, and there are also a large number of hard, open problems.

This is a general talk and no previous knowledge of random walk will be assumed.