

Department of Applied Mathematics and Statistics
The Johns Hopkins University

SEMINAR

Donniell Fishkind
Dept. of Applied Mathematics & Statistics
The Johns Hopkins University

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304 Whitehead Hall
Refreshments: 3:30 p.m.
Seminar: 4:00 p.m.

A GENUS BOUND FOR DIGITAL IMAGE BOUNDARIES

ABSTRACT

A digital image is a three-dimensional array of voxels (volume pixels), each voxel designated as foreground or background. Associated with the image are a foreground and a background graph, whose vertices are contiguous regions of foreground (resp., background) of the same height, and whose edges are contiguous regions of intersection between the foreground (resp., background) vertices.

Let A be a digital image such that its boundary ∂A is a surface, and let $g(\partial A)$ denote the genus of this boundary. Let r_F and r_B denote, respectively, the dimensions of the cycle spaces associated with the foreground and background graphs. We prove that

$$\max\{r_F, r_B\} \leq g(\partial A) \leq r_F + r_B$$

and we show that these bounds are best possible. This result implies the Spherical Homeomorphism Theorem, which is currently used in topology correction of magnetic resonance imaging of the human cerebral cortex.

We also discuss the nature of obstructions that prevent the boundary of a digital image from being a surface, and we present a canonical correction technique.

[This is joint work with Lowell Abrams of the Department of Mathematics at The George Washington University and department colleague Carey Priebe (who is also affiliated with The Johns Hopkins University's Center for Imaging Science).]