Department of Applied Mathematics and Statistics
The Johns Hopkins University

SEMINAR

Jim Fill
Dept. of Applied Mathematics & Statistics
The Johns Hopkins University

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Refreshments: 3:30 p.m.
Seminar: 4:00 p.m.

TRANSFER THEOREMS AND ASYMPTOTIC DISTRIBUTIONAL RESULTS FOR MULTIWAY SEARCH TREES

Abstract

We derive asymptotics of moments and distributions, under the random permutation model on m-ary search trees (fundamental data structures for searching and sorting), for functionals that satisfy recurrence relations of a simple additive form. Many important functionals including the space requirement, internal path length, and the so-called shape functional fall under this framework. The approach is based on establishing *transfer theorems* that link the order of growth of the input into a particular (deterministic) recurrence to the order of growth of the output. The transfer theorems are used in conjunction with the method of moments to establish limit laws. It is shown that (i) for suitably "small" toll sequences we have asymptotic normality if $m \leq 26$ and typically periodic behavior if $m \geq 27$; (ii) for "moderate" toll sequences we have convergence to non-normal distributions if $m \leq m_0$ (where $m_0 \geq 26$) and typically periodic behavior if $m \geq m_0+1$; and (iii) for "large" toll sequences we have convergence to non-normal distributions for all values of m.

Very recent research greatly sharpens understanding of periodic cases. Consider for example the space requirement of m-ary search trees under the random permutation model when $m \geq 27$ is fixed. Chauvin and Pouyanne have shown that the space requirement of an m-ary search tree on n keys equals

$$\mu(n+1) + 2\mathrm{Re}[n^{\lambda_2}\Lambda] + \epsilon_n n^{\mathrm{Re}\lambda_2},$$

where $\mu \in \mathbf{R}$ and $\lambda_2 \in \mathbf{C}$ are certain constants, Λ is a complex-valued random variable, and $\epsilon_n \to 0$ a.s. and in L^2 as $n \to \infty$. Using the elementary but powerful "contraction method," we identify the distribution of Λ. To our knowledge, this is the first application of the contraction method to an oscillatory case.

As time and research progress permit, we will show how this periodic-case result can be extended rather generally to other additive functionals under the random permutation model and, beyond these, to generalized Pólya urn schemes and multi-type branching processes.

(This is joint work with Nevin Kapur of Caltech's Department of Computer Science.)