

Department of Applied Mathematics and Statistics
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STUDENT SEMINAR

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AN INVESTIGATION OF PROXIMITY CATCH DIGRAPHS
IN DELAUNAY TESSELATIONS

ABSTRACT

In this dissertation-based talk, we introduce *proximity catch digraphs* (PCDs) based on two sets (classes) of points, \mathcal{X}_n and \mathcal{Y}_m , in \mathbf{R}^d with $d > 1$, investigate their properties, and present examples in \mathbf{R}^2 for illustrative purposes.

PCDs are a generalization of the class cover catch digraphs (CCCDs) introduced by Priebe et al.; a slight modification of the CCCD yields a special case of PCDs. The PCDs are constructed in terms of the relative positions of members of one class of points, say \mathcal{X}_n , with respect to members of the same class and the Delaunay tessellation of the other class, \mathcal{Y}_m . Our main motivation for introducing PCDs is that a direct extension of the CCCD based on data in \mathbf{R} to data in \mathbf{R}^d with $d > 1$ lacks mathematical tractability of the distribution of the domination number, moments of relative density, and geometry invariance for uniform \mathcal{X}_n in the Delaunay cells constructed using points from \mathcal{Y}_m . We investigate two major aspects of the PCDs, namely, the distribution of the domination number and the relative density. In \mathbf{R}^2 , we analyze these concepts for \mathcal{X}_n in one triangle—the triangle formed by \mathcal{Y}_3 —and then generalize the analyses for \mathcal{X}_n in multiple triangles forming the convex hull of \mathcal{Y}_m for $m > 3$ where Delaunay triangulation of \mathcal{Y}_m is assumed to exist.

Our PCDs make tractable the mathematics in multiple dimensions, thereby enhancing the applicability of the methodology to statistical hypothesis testing and pattern classification. We compute the asymptotic distribution of the domination number of one of the PCDs we introduce. Furthermore, the relative density of the PCDs is shown to be a U -statistic which avails the asymptotic normality of the relative density. Then domination number and relative density are both used to test a type of complete spatial randomness against spatial point patterns of segregation and association. The power of the tests is investigated by using asymptotic efficacy methods such as Pitman asymptotic efficacy, Hodges–Lehman asymptotic efficacy, and asymptotic power function analysis. Finite sample power is analyzed by Monte Carlo simulations. The methods are illustrated in the two-dimensional case, but are applicable to higher dimensions as well as to other types of proximity maps.