

Department of Applied Mathematics and Statistics  
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SEMINAR

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September 9, 2004  
304 Whitehead Hall  
Refreshments: 3:30 p.m.  
Seminar: 4:00 p.m.

SYMMETRY BREAKING AND IMPROVED TAIL PROBABILITY  
BOUNDS FOR THE RANGE STATISTIC

ABSTRACT

Probabilities of the form

$$P \left[ \max_{i=1,\dots,d} X_i - \min_{i=1,\dots,d} X_i \geq c \right]$$

are considered, where the random variables  $X_1, \dots, X_d$  have a known distribution.

A general *abstract tube* approach giving an improved inclusion–exclusion identity for the probability content of a convex polyhedron will be described. Here the term “improved” means that (a) fewer terms are required than in classical inclusion–exclusion, and (b) the bounds obtained from truncation at some depth are sharper. This methodology will be applied in the particular case of the *range polyhedron*

$$\mathcal{P} = \left\{ x \in \mathbf{R}^d : \max_{i=1,\dots,d} x_i - \min_{i=1,\dots,d} x_i \leq c \right\}.$$

This turns out to be an interesting case because this polyhedron has a great deal of symmetry. It will be demonstrated how breaking this symmetry leads to even further improvements.

(This is joint work with Qunming Dong.)