

Department of Applied Mathematics and Statistics  
The Johns Hopkins University

SEMINAR

Gregory L. Eyink  
Dept. of Applied Mathematics & Statistics  
The Johns Hopkins University

March 4, 2004  
304 Whitehead Hall  
Refreshments: 3:30 p.m.  
Seminar: 4:00 p.m.

**RUELLE'S LINEAR RESPONSE FORMULA,  
ENSEMBLE ADJOINT SCHEMES, AND LÉVY FLIGHTS**

ABSTRACT

A traditional subject in statistical physics is the linear response of a molecular dynamical system to changes in an external forcing, e.g. the Ohmic response of an electrical conductor to an applied electric field. For molecular systems the linear response matrices, such as the electrical conductivity, can be represented by “Green–Kubo formulae” as improper time integrals of two-time correlation functions in the system. Recently, mathematical physicist D. Ruelle has extended the Green–Kubo formalism to describe the statistical steady-state response of a general chaotic nonlinear dynamical system to changes in its parameters. This formalism potentially has a number of important applications. For instance, in studies of global warming one wants to calculate the response of climate mean temperature to a change in the atmospheric concentration of various greenhouse gases. In general, a “climate sensitivity” is defined as the linear response of a long-time average to changes in external forces. I’ll show that Ruelle’s linear response formula can be computed by an “ensemble adjoint” technique and that this algorithm is equivalent to a more standard ensemble adjoint method proposed by Lea, Allen, and Haine to calculate climate sensitivities. I’ll give a brief explanation of adjoint algorithms, which are a standard computational method to calculate derivatives of a time-averaged variable simultaneously to several different control parameters. Finally, I’ll present results on numerical implementation of the two ensemble-adjoint methods in the three-variable chaotic Lorenz model and discuss the implications of this example for ensemble-adjoint techniques more generally.

(This talk is based on joint work with two Johns Hopkins colleagues, Thomas Haine in the Department of Earth and Planetary Sciences and Daniel Lea in the Applied Physics Laboratory.)