Department of Applied Mathematics and Statistics
The Johns Hopkins University

THE FIFTH ANNUAL ALAN J. GOLDMAN LECTURE

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304 Whitehead Hall
Refreshments: 3:30 p.m.
Lecture: 4:00 p.m.

DIFFERENTIAL VARIATIONAL INEQUALITIES

Abstract

We introduce and study the class of *differential variational inequalities* (DVIs) in a finite-dimensional Euclidean space. The DVI provides a powerful modeling paradigm for many applied problems in which dynamics, inequalities, and discontinuities are present; examples of such problems include constrained time-dependent physical systems with unilateral constraints, differential Nash games, and hybrid engineering systems with variable structures. The DVI unifies several mathematical problem classes that include ordinary differential equations (ODEs) with smooth and discontinuous right-hand sides, differential algebraic equations (DAEs), dynamic complementarity systems, and evolutionary variational inequalities. We present conditions under which the DVI can be converted, either locally or globally, to an equivalent ODE with a Lipschitz continuous right-hand function. For DVIs that cannot be so converted, we consider their numerical resolution via an Euler time-stepping procedure, which involves the solution of a sequence of finite-dimensional variational inequalities. Borrowing results from differential inclusions (DIs) with upper-semicontinuous, closed, and convex valued multifunctions, we establish the convergence of such a procedure for solving initial-value DVIs. We also present a class of DVIs for which the theory of DIs is not directly applicable, and yet for which similar convergence can be established. Finally, we extend the method to a boundary-value DVI and provide conditions for the convergence of the method.